

Classification and stability of vacua in maximal gauged supergravity

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ABSTRACT: This article presents a systematic study of critical points for the $SL(8, \mathbb{R})$ -type gauging in four dimensional maximal gauged supergravity. We determine all the possible vacua for which the origin of the moduli space becomes a critical point. We formulate a new tool which enables us to find analytically the mass spectrum of the corresponding vacua in terms of eigenvalues of the embedding tensor. When the cosmological constant is nonvanishing, it turns out that many vacua obtained by the dyonic embedding admit a single deformation parameter of the theory, in agreement with the results of the recent paper by Dall'Agata, Inverso and Trigiante [1]. Nevertheless, it is shown that the resulting mass spectrum is independent of the deformation parameter and can be classified according to the unbroken gauge symmetry at the vacua, rather than the underlying gauging. We also show that the generic Minkowski vacua exhibit instability.

KEYWORDS: dS vacua in string theory, Flux compactifications, Superstring Vacua.

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1. Introduction

The maximal supergravity has played a distinguished role in the development of string/M-theory. Although the maximal supergravity fails to describe our realistic chiral world, lots of attentions have been paid to this theory mainly due to the hope of ultraviolet finiteness. Thanks to the high degree of supersymmetry, the particle spectrum is unified into a single supermultiplet and there is no freedom to couple additional matter fields. The only known deformation of maximal supergravity is to gauge the theory by promoting the abelian gauge fields to the nonabelian ones. The gauging procedure gives rise to the scalar potential, as well as the fermionic mass terms. Recently the gauged supergravity theories have been intensively studied in the context of flux compactifications, the gauge/gravity duality and also the condensed matter physics applications.

The original $N = 8$ ungauged supergravity was constructed by Cremmer and Julia via a toroidal compactification of eleven dimensional supergravity [2]. de Wit and Nicolai provided the first example of maximal gauged supergravity by gauging 28 vector fields to have an $SO(8)$ gauge invariance, based on the formalism of T -tensor [3]. This theory has a simple higher dimensional origin, since it is obtained by a dimensional reduction of eleven dimensional supergravity on a seven sphere [4]. Later on, some noncompact gaugings were found to be possible without giving rise to ghost [5]. Subsequently, these types of gaugings have provided a variety of nontrivial vacua. It is then important to explore which types of gaugings are consistently realizable. However, this is not an easy task since viability is sensitive to the choice of (possibly non-semisimple) gauge groups and their embeddings [6, 7]. Even if the consistent gauging is assigned, the extremalization of scalar potential is a formidable task in a general setting, since 70 scalar fields appear in the theory.

Thus, the vacuum hunting so far has been mainly focused upon the truncated sectors where only a few invariant scalars survive. This strategy has been active during the past 25 years [5, 8, 9, 10, 11, 12, 13, 14, 15]. In this traditional approach we need to choose the particular gauging, compute the scalar potential and then scan the moduli space of critical points of the potential. Specifically,

de Sitter (dS) extrema have been found for $\text{SO}(4,4)$ and $\text{SO}(5,3)$ gaugings by confining to the $\text{SO}(p) \times \text{SO}(q)$ invariant scalar. At these vacua spontaneous supersymmetry breakings occur, hence they may be relevant for the early stages of the universe. Although this approach offers a concise way to find vacua, it does not reveal more than the invariant scalars of specific subgroup. For example, despite the fact that all singlet scalars of $\text{SU}(4)^- \subset \text{SO}(8)$ invariant sector are stable, non-singlet scalars do indeed have instabilities [13]. It is therefore desirable to address the systematic survey of viable gaugings, scanning vacua and full stability thereof.

We have recently witnessed two progresses in this line of research. One is the development of a new computational tool to find vacua [16]. A dozen of new critical points have been discovered numerically. Remarkably it was worked out that some nonsupersymmetric vacua are perturbatively stable. These intensive works indicate the possibility of an abundant variety of vacua with potential phenomenological applications. For instance, the anti-de Sitter (AdS) vacua are expected to be dual to the nontrivial conformal fixed points in the dual field theory.

Another development is the formulation based on the embedding tensor [17, 18, 19]. The embedding tensor specifies how to embed the gauge group into the duality group. Using this formalism, all different gaugings can be described in a covariant manner and admissible gaugings can be characterized group-theoretically.

Under these circumstances, Dall’Agata and Inverso utilized the homogeneity of the scalar coset space to determine the complete mass spectrum of 70 scalar fields for some gaugings [20] (see [21] for an early study in half maximal supergravity). Instead of viewing the scalar potential as nonlinear functions of seventy scalar fields, it may be identified as a quadratic function of the embedding tensor. Since the scalar coset $E_{7(7)}/\text{SU}(8)$ is homogeneous, any point can be brought to the origin by the $E_{7(7)}$ isometry, which acts also on the embedding tensor. Hence the critical point of the scalar potential can be mapped to the origin, at the price of varying the embedding tensor. Namely we can explore the possible gaugings, critical points and their mass spectrum at the origin of the scalar manifold, where the governing equations can be analyzed algebraically. See e.g., [22, 23, 24] for some related works by this method.

The aim of this paper is to deepen our understanding of vacuum structure in maximal supergravity. We make a systematic study of vacua and address some issues unresolved in [20]. We give a new tool which enables us to trace analytically the vacuum stability without resorting numerics or annoying diagonalization of 70×70 mass matrices. We conclude that apart from the Minkowski vacua, the mass spectrum is determined by the residual gauge symmetry, rather than the gauging itself. In the meanwhile, the Minkowski vacua are shown to admit intricate mass spectra and possess instabilities in general.

This paper is organized as follows. In the next section we succinctly describe the embedding tensor formalism and fix our notations. Sections 3 and 4 are devoted to the discussion of vacuum classifications and mass spectra. Finally, we conclude with some future prospects in section 5.

2. Maximal gauged supergravity

In this section we will briefly discuss the gauging of maximal supergravity and fix our notations. In the maximal supergravity, the scalar manifold is described by the $E_{7(7)}/\text{SU}(8)$ nonlinear sigma model. The $E_{7(7)}$ acts on the coset representative as isometries, while acts on the gauge fields as global symmetries. We choose a subgroup of $E_{7(7)}$ and promote it to a local symmetry. In order to keep the supersymmetry, this deformation gives rise to a nontrivial scalar potential, by which 70 scalar fields may get stabilized by acquiring mass. For this purpose, the embedding tensor formalism is of help, since it allows one to trace all equations formally in an $E_{7(7)}$ covariant fashion. Refer to the original paper [19] for a more rigorous discussion.

2.1 Embedding tensor formalism

Since the gauge group is a subgroup of $E_{7(7)}$, its generators X_M can be expressed in terms of the generators, t_α , of $E_{7(7)}$ as

$$X_M = \Theta_M^\alpha t_\alpha, \quad (2.1)$$

where $\alpha = 1, \dots, 133$ and $M = 1, \dots, 56$. The gaugings are encoded into the real embedding tensor Θ_M^α belonging to the $\mathbf{56} \times \mathbf{133}$ representation of $E_{7(7)}$. It specifies which generators of the duality group to be chosen as generators of gauge group. A major advantage of adopting the embedding tensor is that it allows us to keep the entire formulation in a duality covariant way. In terms of X_M the symmetry can be made local by introducing gauge covariant derivative,

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - g A_\mu^M X_M, \quad (2.2)$$

where g is the coupling constant.

The embedding tensor must satisfy linear and quadratic constraints in order to ensure the consistent gaugings. The quadratic constraint equation requires that the embedding tensor should be invariant under the gauge group,

$$C_{MN}^\alpha := f_{\beta\gamma}^\alpha \Theta_M^\beta \Theta_N^\gamma + (t_\beta)_N^P \Theta_M^\beta \Theta_P^\alpha = 0, \quad (2.3)$$

where $f_{\alpha\beta}^\gamma$ denotes the structure constants of $E_{7(7)}$, i.e., $[t_\alpha, t_\beta] = f_{\alpha\beta}^\gamma t_\gamma$. Equation (2.3) implies the closure condition $[X_M, X_N] = -X_{MN}^P X_P$ of the gauge algebra. It should be stressed that (2.3) involves a nontrivial information upon the symmetrization in (M, N) .

In addition to the quadratic relation, the supersymmetry imposes the linear constraint upon the embedding tensor. The concrete form of this constraint depends on the spacetime dimensionality and supercharges. In the present case, the embedding tensor is subject to the following restriction [18, 19],

$$(t_\alpha)_M^N \Theta_N^\alpha = 0, \quad (t_\beta t^\alpha)_M^N \Theta_N^\beta = -\frac{1}{2} \Theta_M^\alpha. \quad (2.4)$$

Here and in what follows, the indices α, β, \dots are raised and lowered by the $E_{7(7)}$ Cartan-Killing metric $\eta_{\alpha\beta} = \text{Tr}(t_\alpha t_\beta)$. Although the embedding tensor *a priori* takes values in the tensor product $\mathbf{56} \times \mathbf{133} = \mathbf{56} + \mathbf{912} + \mathbf{6480}$, the above constraint amounts to requiring the embedding tensor to belong to the $\mathbf{912}$ representation [18, 19].¹ From equation (2.4) one may derive $X_{(MNP)} = X_{M[NP]} = X_{MN}^M = 0$, namely, the gauge group must be unimodular. Each solution to the above set of constraints gives rise to a viable gauging.

In the present paper, we are interested in a gauge group embedded into the standard $\text{SL}(8, \mathbb{R})$ subgroup of $E_{7(7)}$. The branching rules into $\text{SL}(8, \mathbb{R})$ representations are

$$\mathbf{56} \rightarrow \mathbf{28} + \mathbf{28}', \quad \mathbf{133} \rightarrow \mathbf{63} + \mathbf{70}, \quad \mathbf{912} \rightarrow \mathbf{36} + \mathbf{420} + \mathbf{36}' + \mathbf{420}'. \quad (2.5)$$

Since the embedding tensor lives in $\mathbf{912}$ representation, the relevant branchings are given by

| | | |
|-----------|-----------------|-------------------|
| | 28 | 28' |
| 63 | 36 + 420 | 36' + 420' |
| 70 | 420' | 420 |

(2.6)

The representations $\mathbf{420}$ and $\mathbf{420}'$ appear not only in representations arising from the adjoint representation $\mathbf{63}$ of $\text{SL}(8, \mathbb{R})$, but also in those coming from $\mathbf{70}$. Hence, for the embeddings in

¹If one gauges the trombone symmetry, the $\mathbf{56}$ representations can be excited [25].

$\text{SL}(8, \mathbb{R})$, the embedding tensor has to belong to **36** and/or **36'**, on which we will concentrate in the rest of the paper.

The scalar potential arises from the $O(g^2)$ corrections of supersymmetry transformations. In terms of X_M , it is given by [19]

$$V = \frac{g^2}{672} (X_{MN}{}^R X_{PQ}{}^S M^{MP} M^{NQ} M_{RS} + 7 X_{MN}{}^Q X_{PQ}{}^N M^{MP}) , \quad (2.7)$$

where M_{MN} is a real and symmetric matrix with the inverse M^{MN} and defined by

$$M = L \cdot {}^T L , \quad M_{MN} = (M)_{MN} . \quad (2.8)$$

Here $L = L(\phi)$ is the coset representative in the $\text{Sp}(56, \mathbb{R})$ representation. From the higher dimensional point of view, the four dimensional scalar potential encodes the internal geometry and the flux contributions. For generic gaugings, the potential is unbounded both below and above, and fails to have any extrema.

For later convenience, let us recapitulate some coset representations. Cremmer and Julia introduced the $\text{Usp}(56)$ representation, in which the diagonal element of $E_{7(7)}$ algebra is $\text{SU}(8)$ [2]. In the $\text{Usp}(56)$ representation the coset representatives take the form,

$$L(\phi)_{\underline{M}}{}^{\underline{N}} = \exp \begin{pmatrix} 0 & \phi_{ijkl} \\ \phi^{ijkl} & 0 \end{pmatrix} , \quad \phi_{ijkl} = \phi_{[ijkl]} = \eta(\star \bar{\phi})_{ijkl} , \quad (2.9)$$

where the underlined indices refer to **28** + $\overline{\text{28}}$ of $\text{SU}(8)$, and $\eta = \pm 1$ corresponds to the chirality of the spinor representation of $\text{SO}(8)$ below. Here i, j, \dots are **8** and $\bar{\text{8}}$ of $\text{SU}(8)$, and are raised and lowered via complex conjugation, as usual. The change of basis can be done via gamma matrices in the real Weyl spinor representation of $\text{SO}(8)$,

$$L_{\underline{M}}{}^{\underline{N}} = \mathcal{S}_{\underline{M}}{}^P L_P{}^Q (\mathcal{S}^{-1})_Q{}^{\underline{N}} , \quad \mathcal{S}_{\underline{M}}{}^N = \frac{i}{4\sqrt{2}} \begin{pmatrix} \Gamma_{ij}{}^{ab} & i\Gamma_{ijab} \\ \Gamma^{ijab} & -i\Gamma^{ij}{}_{ab} \end{pmatrix} , \quad (2.10)$$

where $(\Gamma_{ij})^{ab} = (\Gamma^{ab})_{ij} =: \Gamma_{ij}^{ab}$, and there is no need to distinguish their upper and lower indices. In particular, we denote by \mathcal{V} to describe the coset representative in a mixed basis,

$$\mathcal{V}_M{}^{\underline{N}} = L_M{}^P (\mathcal{S}^{-1})_P{}^{\underline{N}} . \quad (2.11)$$

2.2 Mass matrix

The seventy scalars parametrize the homogeneous (and moreover symmetric) coset space $E_{7(7)}/\text{SU}(8)$. The homogeneity means that every point on the (Riemannian) manifold can be mapped into any other point via a global transformation (isometry). In other words, the manifold admits the transitive group of motions.

What is important here is that the scalar potential is invariant under the simultaneous transformations of the coset representative and of the embedding tensor. Indeed, the potential depends on a single tensorial combination $L^{-1}\Theta$. To see this, let us define

$$\tilde{\Theta}_M{}^\alpha t_\alpha := (L^{-1})_M{}^N \Theta_N{}^\alpha L^{-1} t_\alpha L . \quad (2.12)$$

This is the analogue of T -tensor in the $\text{Sp}(56, \mathbb{R})$ representation. In terms of $\tilde{\Theta}_M{}^\alpha$, the potential (2.7) can be expressed as

$$V = \frac{g^2}{672} \tilde{\Theta}_M{}^\alpha \tilde{\Theta}_N{}^\beta (\delta_{\alpha\beta} + 7\eta_{\alpha\beta}) , \quad (2.13)$$

where

$$\text{Tr}(t_\alpha t_\beta^\dagger) = \delta_{\alpha\beta}, \quad \text{Tr}(t_\alpha t_\beta) = \eta_{\alpha\beta}. \quad (2.14)$$

In this form, one notices that the potential depends (quadratically) only on $\tilde{\Theta}_M^\alpha$, as we desired to show. Since any point on the scalar manifold can be mapped to any other point, the optimal setup is to move the critical point to the origin ($\phi_{ijkl} = 0$), where $L(\phi) = \mathbb{I}_{56}$. At the origin, the extremum condition amounts to the quadratic conditions on Θ_M^α .

To take the first variation of the potential, we first note that every variation of the coset representative can be written in a form of an $E_{7(7)}$ transformation from the right. In the $\text{Sp}(56, \mathbb{R})$ representation we have

$$\partial_\rho L = -L t_\rho, \quad (2.15)$$

where index ρ refers exclusively to 70 noncompact elements of $\mathfrak{e}_{7(7)}$. This implies

$$\partial_\rho L^{-1} = t_\rho L^{-1}, \quad \partial_\rho M = -L(t_\rho + {}^T t_\rho)^T L, \quad \partial_\rho M^{-1} = ({}^T L)^{-1}(t_\rho + {}^T t_\rho)L^{-1}. \quad (2.16)$$

Because of the relation

$$\partial_\rho(\tilde{\Theta}_M^\alpha t_{\alpha N}^P) = t_{\rho M}^Q \tilde{\Theta}_Q^\alpha t_{\alpha N}^P + \tilde{\Theta}_M^\alpha f_{\rho\alpha}{}^\gamma t_{\gamma N}^P, \quad (2.17)$$

the first derivative of V is obtained as

$$\partial_\rho V = \frac{g^2}{336} \left[t_{\rho M}^N \tilde{\Theta}_M^\alpha \tilde{\Theta}_N^\beta (\delta_{\alpha\beta} + 7\eta_{\alpha\beta}) + \tilde{\Theta}_M^\alpha \tilde{\Theta}_M^\beta f_{\rho\beta}{}^\gamma \delta_{\alpha\gamma} \right]. \quad (2.18)$$

At the origin, $\partial_\rho V = 0$ imposes a quadratic restriction upon Θ_M^α , which should be combined to be solved with (2.3) and (2.4). It turns out that we can scan the critical points and underlying gaugings at the same time, as demonstrated in [20].

We can furthermore discuss the mass spectrum at the same time. By virtue of (2.15)–(2.18), we can likewise obtain the second derivatives of the potential

$$\begin{aligned} \partial_\sigma \partial_\rho V = \frac{g^2}{336} \left[(t_\rho + {}^T t_\rho)_M^N t_{\sigma M}^P \tilde{\Theta}_P^\alpha \tilde{\Theta}_N^\beta (\delta_{\alpha\beta} + 7\eta_{\alpha\beta}) \right. \\ \left. + \tilde{\Theta}_M^\alpha \tilde{\Theta}_M^\beta (f_{\rho\alpha}{}^\gamma f_{\sigma\beta}{}^\delta \delta_{\gamma\delta} + (f_{(\rho} f_{\sigma)})_\alpha{}^\gamma \delta_{\beta\gamma}) \right. \\ \left. + 2(t_{\rho M}^N f_{\sigma\alpha}{}^\gamma + t_{\sigma M}^N f_{\rho\alpha}{}^\gamma) \delta_{\beta\gamma} \tilde{\Theta}_M^{(\alpha} \tilde{\Theta}_N^{\beta)} \right]. \end{aligned} \quad (2.19)$$

In order to reduce (2.19) to a more tractable form, we rely on the observation $[{}^T t_\alpha, t_\beta] \in \mathfrak{e}_{7(7)}$, which suggests that there exist constants $c_{\alpha\beta}{}^\gamma$ such that $[{}^T t_\alpha, t_\beta] = c_{\alpha\beta}{}^\gamma t_\gamma$. Applying the Jacobi identity to $({}^T t_\alpha, t_\beta, t_\gamma)$, we have

$$c_{(\rho\sigma)}{}^\gamma f_{\gamma\alpha}{}^\delta \delta_{\beta\delta} = -c_{\beta(\rho}{}^\gamma c_{\sigma)\alpha}{}^\gamma - f_{\alpha(\rho}{}^\gamma f_{\sigma)\beta}{}^\delta \delta_{\gamma\delta}. \quad (2.20)$$

Using this relation, a simple computation shows that (2.19) can be cast into

$$\partial_\rho \partial_\sigma V = (M^2)_{\rho\sigma} + \frac{1}{2} c_{(\rho\sigma)}{}^\gamma \partial_\gamma V, \quad (2.21)$$

where M^2 describes the mass matrix at the extrema,

$$\begin{aligned} (M^2)_{\rho\sigma} := \frac{g^2}{168} \left[(s_{(\rho} s_{\sigma)})_M^N \text{Tr}(X_M {}^T X_N + 7X_M X_N) + 2(s_{(\rho})_M^N \text{Tr}(s_{\sigma)} [X_{(M}, {}^T X_{N)})] \right. \\ \left. - \text{Tr}([s_{(\rho}, X_M] [s_{\sigma)}, {}^T X_M]) \right]. \end{aligned} \quad (2.22)$$

Here we have defined 56×56 symmetric matrices,

$$s_\rho := \frac{1}{2} (t_\rho + {}^T t_\rho). \quad (2.23)$$

3. Electric gaugings

We first discuss the case in which the gauge group is contained in the $\text{SL}(8, \mathbb{R})$ electric frame. This is the simplest setup where the relation to the flux compactification is clear [26, 27, 28]. This type of gaugings corresponds to the $\text{CSO}(p, q, r)$ gaugings.

Since the embedding tensor is described by the **36'** representation of $\text{SL}(8, \mathbb{R})$ [18] as

$$\Theta_{ab}{}^c{}_d = \delta^c_{[a} \theta_{b]d}, \quad \theta_{ab} = \theta_{(ab)}, \quad a, b, \dots = 1, \dots, 8, \quad (3.1)$$

the gauge structure constants X_M of $\text{CSO}(p, q, r)$ are given by

$$X_M = (X_\Lambda, X^\Sigma) = (X_{[ab]}, 0), \quad X_{[ab]} = \begin{pmatrix} X_{[ab][cd]}{}^{[ef]} & 0 \\ 0 & X_{[ab]}{}^{[cd]}{}_{[ef]} \end{pmatrix}, \quad (3.2)$$

where

$$X_{[ab][cd]}{}^{[ef]} = \delta_{[a}^{[e} \theta_{b][c} \delta_{d]}^{f]}, \quad X_{[ab]}{}^{[cd]}{}_{[ef]} = -\delta_{[a}^{[c} \theta_{b][e} \delta_{f]}^{d]}. \quad (3.3)$$

In this case the quadratic constraint is automatically satisfied. Thus any symmetric tensor θ_{ab} defines a consistent gauging even if it is noninvertible.

From (2.7) and (3.1) the potential at the extrema is given by

$$V_c = \frac{1}{8} g^2 \left[\frac{1}{4} \text{Tr}(\theta^2) - \frac{1}{8} (\text{Tr}\theta)^2 \right]. \quad (3.4)$$

In our present normalization, V_c is equivalent to the cosmological constant.

3.1 Vacua

In the electric gauging case, the origin of the scalar coset corresponds to the critical point if the following relation holds [20]

$$2\theta^2 - \theta \text{Tr}\theta = 2v \mathbb{I}_8, \quad v := 4g^{-2} V_c. \quad (3.5)$$

Note that the extremum condition is invariant under $\theta \rightarrow P^{-1} \theta P$. Hence we can confine ourselves to the diagonal θ by taking P as an orthogonal matrix. Since the 8×8 matrix θ obeys a quadratic equation (3.5), its eigenvalues λ_i ($i = 1, 2$) should satisfy

$$\lambda_i^2 - x \lambda_i - v = 0, \quad x := \frac{1}{2} \text{Tr}(\theta). \quad (3.6)$$

Let n_i ($\sum_i n_i = 8$) denote the degeneracy of eigenvalue λ_i . Then the extremum condition translates into

$$\sum_i (n_i - 2) \lambda_i = 0. \quad (3.7)$$

Since we have

$$V_c = -\frac{1}{4} g^2 \lambda_1 \lambda_2, \quad (3.8)$$

the potential vanishes for $n_1 = 2$ or $n_2 = 2$, in which case the θ tensor is noninvertible. One also finds that the potential is invariant under $\lambda_i \rightarrow -\lambda_i$ and $\lambda_1 \leftrightarrow \lambda_2$, thereby these cases correspond to the same vacua.

It is observed that equations (3.4) and (3.5) are invariant under the rescaling $\theta \rightarrow e^\alpha \theta$ with $g \rightarrow e^{-\alpha} g$ ($\alpha \in \mathbb{R}$). Using this freedom, we are free to set $\det \theta = \pm 1$ for $n_i \neq 2$, whereas for $n_1 = 2$ ($n_2 = 2$) we can choose λ_1 (λ_2) to take any nonvanishing value. Under these conditions, it turns out that the extrema in the electric gaugings are exhausted by table 1 of reference [20].

3.2 Mass spectrum

We now move on to the main part of this paper and determine analytically the full mass spectrum of 70 scalars. Let $s_\rho = (t_\rho + {}^T t_\rho)/2$ decompose into

$$s_\rho = \begin{pmatrix} s_{\rho[ab]}^{[cd]} & s_{\rho[abcd]} \\ s_{\rho[abcd]} & s_{\rho}^{[cd]}{}_{[ab]} \end{pmatrix}, \quad (3.9)$$

where

$$s_{\rho[ab]}^{[cd]} = -s_{\rho}^{[cd]}{}_{[ab]} = 2(S_\rho)_{[a}{}^{[c}\delta_{b]}{}^{d]}, \quad (s_\rho)_{[abcd]} = (s_\rho)^{[abcd]} = (U_\rho)_{[abcd]}. \quad (3.10)$$

Each of real tensors (S, U) has 35 components and satisfies

$$S = {}^T S, \quad \text{Tr}(S) = 0, \quad U = \star U. \quad (3.11)$$

Substituting (3.2) and (3.9) into (2.22), we are led to

$$M^2 = M_{(1)}^2(\theta) + M_{(2)}^2(\theta), \quad (3.12)$$

with

$$M_{(1)}^2(\theta) = \frac{1}{8}g^2 [-\text{Tr}(\theta)\text{Tr}(S^2\theta) - [\text{Tr}(\theta S)]^2 + 2\text{Tr}(S^2\theta^2) + 2\text{Tr}(S\theta S\theta)], \quad (3.13)$$

$$M_{(2)}^2(\theta) = \frac{1}{8}g^2 \left[-U_{[ab][cd]}^2 \theta_{ac}\theta_{bd} + \frac{1}{24}U \cdot U \text{Tr}(\theta^2) \right]. \quad (3.14)$$

Here we have introduced the abbreviation

$$U \cdot U = U_{abcd}U_{abcd}, \quad (U^2)_{[ab][cd]} = U_{abef}U_{cdef}, \quad (U^2)_{ab} = U_{acde}U_{bcde} = \frac{1}{8}U \cdot U \delta_{ab}, \quad (3.15)$$

where the final expression follows from the self-duality of U .

We now split the matrix S into n_1 and n_2 blocks

$$S = \begin{pmatrix} A_{11} & A_{12} \\ {}^T A_{12} & A_{22} \end{pmatrix}, \quad \theta = \begin{pmatrix} \lambda_1 \mathbb{I}_{n_1} & \\ & \lambda_2 \mathbb{I}_{n_2} \end{pmatrix}, \quad (3.16)$$

and define

$$A_{11} = \frac{1}{n_1}\text{Tr}(A_{11})\mathbb{I}_{n_1} + \hat{A}_1, \quad A_{22} = -\frac{1}{n_2}\text{Tr}(A_{11})\mathbb{I}_{n_2} + \hat{A}_2, \quad (3.17)$$

where \hat{A}_1 and \hat{A}_2 are trace-free parts of A_{11} and A_{22} , respectively.

In order to achieve the correct mass spectrum we need to canonically normalize the scalar kinetic function. According to (2.10), the fluctuations of scalar fields $\delta\phi_{ijkl}$ are given by

$$2S_{[a}{}^{[c}\delta_{b]}{}^{d]} + iU_{abcd} = \frac{1}{16}\Gamma_{ab}^{ij}\Gamma_{cd}^{kl}\delta\phi_{ijkl}, \quad (3.18)$$

Then the scalar kinetic term reads

$$\frac{1}{12}\mathcal{P}_{\mu ijkl}\mathcal{P}^{\mu ijkl} = \frac{1}{12}|\partial_\mu\phi_{ijkl}|^2 = \frac{1}{2}\text{Tr}((\partial S)^2) + \frac{1}{12}\partial U \cdot \partial U. \quad (3.19)$$

It follows that

$$\frac{1}{2}\text{Tr}((\partial S)^2) = \frac{1}{2} \left[\frac{8}{n_1 n_2} (\partial \text{Tr} A_{11})^2 + \text{Tr}((\partial \hat{A}_1)^2) + \text{Tr}((\partial \hat{A}_2)^2) + 2\text{Tr}(\partial {}^T A_{12} \partial A_{12}) \right]. \quad (3.20)$$

With reference to (3.13) and (3.16), the mass matrix $M_{(1)}^2$ can be expressed in terms of fields $(\text{Tr}(A_{11}), \hat{A}_1, \hat{A}_2, A_{12})$. The canonical mass eigenvalues can be read off in such a way that each coefficient of these fields agrees with (3.20), thereby

$$M_{(1)}^2 = \frac{8}{n_1 n_2} m_{0(\mathbf{1}, \mathbf{1})}^2 \text{Tr}(A_{11})^2 + m_{1(\mathbf{N}_1, \mathbf{1})}^2 \text{Tr}(\hat{A}_1^2) + m_{2(\mathbf{1}, \mathbf{N}_2)}^2 \text{Tr}(\hat{A}_2^2) + 2m_{*(\mathbf{n}_1, \mathbf{n}_2)}^2 \text{Tr}(^T A_{12} A_{12}), \quad (3.21)$$

where

$$N_1 = \frac{1}{2}(n_1 - 1)(n_1 + 2), \quad N_2 = \frac{1}{2}(n_2 - 1)(n_2 + 2), \quad (3.22)$$

and

$$m_{0(\mathbf{1}, \mathbf{1})}^2 = \frac{g^2}{64} [2n_2(2 - n_1)\lambda_1^2 + 2n_1(2 - n_2)\lambda_2^2 - (n_1 - n_2)^2 \lambda_1 \lambda_2], \quad (3.23a)$$

$$m_{1(\mathbf{N}_1, \mathbf{1})}^2 = \frac{g^2}{8} \lambda_1 [(4 - n_1)\lambda_1 - n_2 \lambda_2], \quad (3.23b)$$

$$m_{2(\mathbf{1}, \mathbf{N}_2)}^2 = \frac{g^2}{8} \lambda_2 [(4 - n_2)\lambda_2 - n_1 \lambda_1], \quad (3.23c)$$

$$m_{*(\mathbf{n}_1, \mathbf{n}_2)}^2 = \frac{g^2}{16} (\lambda_1 + \lambda_2) [(2 - n_1)\lambda_1 + (2 - n_2)\lambda_2] = 0. \quad (3.23d)$$

At the last equality we have used the stationary point condition (3.7). It follows that the A_{12} field is always massless. Boldface letters in the subscript denote the representations of $\text{SO}(n_1) \times \text{SO}(n_2)$. This notation manifests multiplicities explicitly, i.e., $m_{(\mathbf{k}_1, \mathbf{k}_2)}^2$ represents the mass spectrum for fields with $k_1 k_2$ degeneracies. Note that fluctuations of $\text{Tr}(A_{11})$ and A_{12} exist for $n_1 n_2 > 0$, while \hat{A}_1 (\hat{A}_2) exists for $n_1 > 1$ ($n_2 > 1$).

When $n_1 \neq 2, 6$, the cosmological constant is nonvanishing. So we can normalize the mass spectra in a unit of the cosmological constant (3.8) and obtain a more comprehensive form

$$m_{0(\mathbf{1}, \mathbf{1})}^2 = -2V_c, \quad m_{1(\mathbf{N}_1, \mathbf{1})}^2 = \frac{4V_c}{n_1 - 2}, \quad m_{2(\mathbf{1}, \mathbf{N}_2)}^2 = \frac{4V_c}{n_2 - 2}. \quad (3.24)$$

Whereas, for $n_1 = 2$ we have

$$m_{0(\mathbf{1}, \mathbf{1})}^2 = m_{2(\mathbf{1}, \mathbf{20})}^2 = m_{*(\mathbf{2}, \mathbf{6})}^2 = 0, \quad m_{1(\mathbf{2}, \mathbf{1})}^2 = \frac{1}{4} g^2 \lambda_1^2. \quad (3.25)$$

The $n_1 = 6$ case can be deduced similarly.

Let us turn to determine the mass spectrum of pseudoscalars U . We decompose the eight indices into n_1 and n_2 blocks,

$$S_1 = \{1, \dots, n_1\}, \quad S_2 = \{n_1 + 1, \dots, n_1 + n_2\}. \quad (3.26)$$

Let ℓ be a non-negative integer taking values in the range $0 \leq \ell \leq 4$, $0 \leq 4 - \ell \leq n_2$. Then the basis of antisymmetric four-form is labeled by pairs I_1, I_2 , where I_1 (I_2) is a set of ℓ ($4 - \ell$) indices belonging to S_1 (S_2). For any four-form Z_{abcd} , we find

$$\theta^r_{[a} \theta^s_b Z_{cd]rs} = \frac{1}{12} [\ell(\ell - 1)\lambda_1^2 + 2\ell(4 - \ell)\lambda_1 \lambda_2 + (4 - \ell)(3 - \ell)\lambda_2^2] Z_{abcd}, \quad (3.27)$$

from which we are led to

$$(1 + \star)\theta^r_{[a} \theta^s_b U_{cd]rs} = 2\mu_\ell U_{abcd}, \quad (3.28)$$

where

$$\begin{aligned} \mu_\ell = \frac{1}{24} & \left[\ell(\ell-1)\lambda_1^2 + 2\ell(4-\ell)\lambda_1\lambda_2 + (4-\ell)(3-\ell)\lambda_2^2 + (n_1-\ell)(n_1-\ell-1)\lambda_1^2 \right. \\ & \left. + 2(n_1-\ell)(4-n_1+\ell)\lambda_1\lambda_2 + (4-n_1+\ell)(3-n_1+\ell)\lambda_2^2 \right]. \end{aligned} \quad (3.29)$$

Then the fluctuation mode of U is labeled by a non-negative integer ℓ satisfying

$$n_1 \leq 2\ell \leq \min(2n_1, 8). \quad (3.30)$$

Since the kinetic term of U is given by $(1/12)\partial U_{abcd}\partial U_{abcd}$, the normalized mass eigenvalue $m_{[\ell]}$ reads

$$M_{(2)}^2 = \frac{1}{6}m_{[\ell]}^2 U \cdot U, \quad (3.31)$$

where

$$\begin{aligned} m_{[\ell]}^2 = \frac{g^2}{32} & \left[\{n_1 - \ell(\ell-1) - (n_1-\ell)(n_1-\ell-1)\}\lambda_1^2 - 2\{\ell(4-\ell) + (n_1-\ell)(4+\ell-n_1)\}\lambda_1\lambda_2 \right. \\ & \left. + \{n_2 - (4-\ell)(3-\ell) - (4+\ell-n_1)(3+\ell-n_1)\}\lambda_2^2 \right], \end{aligned} \quad (3.32)$$

with multiplicities

$$2\ell > n_1 \quad : \quad n_1 C_\ell \times_{8-n_1} C_{4-\ell}, \quad 2\ell = n_1 \quad : \quad \frac{1}{2}n_1 C_{n_1/2} \times_{8-n_1} C_{4-n_1/2}. \quad (3.33)$$

For $n_1 \neq 2, 6$, equation (3.32) simplifies in a unit of the cosmological constant to

$$m_{[\ell]}^2 = \frac{2[2\ell^2 - 2n_1\ell + (n_1-2)^2]}{(n_1-6)(n_1-2)} V_c. \quad (3.34)$$

For $n_1 = 2$ i.e., $\lambda_2 = 0$, we have

$$m_{[\ell=1](\times 20)}^2 = m_{(\mathbf{2}, \mathbf{20})+}^2 = \frac{1}{16}g^2\lambda_1^2, \quad m_{[\ell=2](\times 15)}^2 = m_{(\mathbf{1}, \mathbf{15})}^2 = 0, \quad (3.35)$$

where we have denoted multiplicities and representations in the subscript, and “+” stands for the self-duality. As argued in the next subsection, these massless modes have nothing to do with the Nambu-Goldstone bosons.

We are now in a position to discuss critical points and mass spectra in the electric gauging. Our classification exhaustively recovers the list of critical points found by Dall’Agata-Inverso [20], which we summarize in table 3.2. Our analytic expressions of mass spectra are in perfect agreement with the reference [20], in which mass eigenvalues may have been obtained by diagonalization of 70×70 mass matrix. In our method it is obvious which parts of ϕ_{ijkl} belong to scalars (S field) and pseudoscalars (U field). Moreover, our formulation makes it clear that the mass spectrum is simply specified group-theoretically by multiplicities. In particular, it immediately turns out that the dS vacua necessarily have unstable mode of $m_0^2 = -2V_c$, arising from the trace part of n_1 -block [see (3.24)]. For $\text{SO}(5, 3)$ and $\text{SO}(4, 4)$ gaugings, this mode corresponds to the $\text{SO}(5) \times \text{SO}(3)$ and $\text{SO}(4) \times \text{SO}(4)$ invariant scalar, respectively [10, 11, 12].

With the exception of Minkowski vacua, we have normalized the mass eigenvalues by the cosmological constant. This normalization is intuitively clear since it measures the curvature of potential. Then, inspection of (3.24) and (3.34) reveals that the mass spectrum is determined only by the remaining gauge symmetry at the vacua (i.e., n_i only), rather than the gauging (actual value of λ). This statement will become more persuasive when we look into the dyonic case in the subsequent section, where several vacua of different gaugings can have the same mass spectra.

| Gauging | G_{reg} | Λ | m_S^2 | m_U^2 | SUSY |
|--------------|----------------------|-----------|---|---|---------|
| SO(4, 4) | SO(4) \times SO(4) | dS | $-2_{(\times 1)}, 2_{(\times 18)}, 0_{(\times 16)}$ | $2_{(\times 18)}, 1_{(\times 16)}, -2_{(\times 1)}$ | none |
| SO(5, 3) | SO(5) \times SO(3) | dS | $-2_{(\times 1)}, \frac{4}{3}_{(\times 14)}, 4_{(\times 5)}, 0_{(\times 15)}$ | $2_{(\times 30)}, -\frac{2}{3}_{(\times 5)}$ | none |
| SO(8) | SO(7) | AdS | $2_{(\times 1)}, -\frac{4}{5}_{(\times 27)}, 0_{(\times 7)}$ | $-\frac{2}{5}_{(\times 35)}$ | none |
| SO(8) | SO(8) | AdS | $-\frac{2}{3}_{(\times 35)}$ | $-\frac{2}{3}_{(\times 35)}$ | $N = 8$ |
| CSO(2, 0, 6) | SO(2) | Mink. | $0_{(\times 33)}, 2_{(\times 2)}$ | $0_{(\times 15)}, \frac{1}{2}_{(\times 20)}$ | none |

Table 1: Mass spectrum for electric gaugings. For the (A)dS vacua, we normalized by the absolute value of the cosmological constant. For the CSO(2, 0, 6) gauging, we take a normalization $g\lambda_1 = 2\sqrt{2}$.

3.3 Spontaneous symmetry breaking

As we have seen above, some scalar fields turn out to be massless. In this section we discuss the Higgs mechanism for the $\text{SL}(8, \mathbb{R})$ electric vacua in order to identify Nambu-Goldstone directions.

Taking the mixed coset representative (2.11), we define

$$\mathcal{Q}_{Mij}{}^{kl} = i\Omega^{NP}\mathcal{V}_{Nij}X_{MP}{}^Q\mathcal{V}_Q{}^{kl}, \quad \mathcal{P}_{Mijkl} = i\Omega^{NP}\mathcal{V}_{Nij}X_{MP}{}^Q\mathcal{V}_Q{}^{kl}, \quad (3.36)$$

where $\Omega^{MN} = \begin{pmatrix} \mathbb{O} & \mathbb{I} \\ -\mathbb{I} & \mathbb{O} \end{pmatrix}$ is the standard metric of $\text{Sp}(56, \mathbb{R})$. These tensors satisfy

$$\mathcal{P}_M{}^{ijkl} = \frac{\eta}{24}\epsilon^{ijklmnpq}\mathcal{P}_{Mnpq}, \quad \mathcal{Q}_{Mij}{}^{kl} = \delta_{[i}^{[k}\mathcal{Q}_{Mj]}{}^{l]}, \quad \mathcal{Q}_M{}^i{}_j = -\mathcal{Q}_{Mj}{}^i, \quad \mathcal{Q}_{Mi}{}^i = 0. \quad (3.37)$$

In terms of the T -tensor²

$$T_{\underline{MN}}{}^{\underline{P}} := \frac{1}{2}(\mathcal{V}^{-1})_{\underline{M}}{}^M(\mathcal{V}^{-1})_{\underline{N}}{}^N\mathcal{V}_P{}^{\underline{P}}X_{MN}{}^P, \quad (3.38)$$

\mathcal{P}_M and \mathcal{Q}_M are expressed as

$$T_k{}^{lij} = \frac{3}{4}i\Omega^{MN}\mathcal{Q}_{Mk}{}^l\mathcal{V}_N{}^{ij}, \quad T_{klmn}{}^{ij} = \frac{1}{2}i\Omega^{MN}\mathcal{P}_{Mklmn}\mathcal{V}_N{}^{ij}. \quad (3.39)$$

These components of the T -tensor are related to the $\text{SU}(8)$ connections and tensors, as shown below. Let us introduce the $\text{SU}(8)$ covariant derivative by

$$\mathcal{D}_\mu\mathcal{V}_M{}^{ij} = \partial_\mu\mathcal{V}_M{}^{ij} - \mathcal{Q}_{\mu k}{}^{ij}\mathcal{V}_M{}^{kl} - gA_\mu{}^P X_{PM}{}^N\mathcal{V}_N{}^{ij}, \quad (3.40)$$

where \mathcal{Q}_μ is an $\text{SU}(8)$ connection, satisfying

$$\mathcal{Q}_{\mu ij}{}^{kl} = \delta_{[i}^{[k}\mathcal{Q}_{\mu j]}{}^{l]}, \quad \mathcal{Q}_{\mu i}{}^j = -\mathcal{Q}_{\mu}{}^j{}_i, \quad \mathcal{Q}_{\mu i}{}^i = 0. \quad (3.41)$$

The $\text{SU}(8)$ covariant derivative is subjected to the restriction $\Omega^{MN}\mathcal{V}_{Mij}\mathcal{D}_\mu\mathcal{V}_N{}^{kl} = 0$, giving

$$\mathcal{Q}_{\mu i}{}^j = \frac{2}{3}i(\mathcal{V}_{\Lambda ik}\partial_\mu\mathcal{V}^{\Lambda jk} - \mathcal{V}^{\Lambda}_{ik}\partial_\mu\mathcal{V}_\Lambda{}^{jk}) - gA_\mu{}^M\mathcal{Q}_{Mi}{}^j. \quad (3.42)$$

Similarly, we can define an $\text{SU}(8)$ covariant tensor

$$\mathcal{P}_{\mu ijkl} = i\Omega^{MN}\mathcal{V}_{Mij}\mathcal{D}_\mu\mathcal{V}_{Nkl}, \quad \mathcal{P}_\mu{}^{ijkl} = \frac{\eta}{24}\epsilon^{ijklmnpq}\mathcal{P}_{\mu mnpq}, \quad (3.43)$$

²Note that the prefactor 1/2 in (3.39) is necessary to derive (3.39). The original paper [19] seems to have a typo.

which specifies the scalar kinetic term (3.19). Then it can be shown that

$$\mathcal{P}_{\mu ijkl} = i(\mathcal{V}_{\Lambda ij} \partial_\mu V_{\Lambda kl} - \mathcal{V}_{\Lambda ij}^\Lambda \partial_\mu \mathcal{V}_{\Lambda kl}) - g A_\mu^M \mathcal{P}_{Mijkl}. \quad (3.44)$$

Let us now take a base point O in the moduli space of scalar manifold and employ the following coset representative

$$\mathcal{V} = \mathcal{V}(O) \exp \begin{pmatrix} 0 & \phi \\ \bar{\phi} & 0 \end{pmatrix}, \quad \phi = (\phi_{ijkl}), \quad \bar{\phi} = \star \phi = \eta(\phi^{ijkl}). \quad (3.45)$$

In this gauge, we obtain

$$\mathcal{Q}_{\mu i}^j = -g A_\mu^M \mathcal{Q}_{Mi}^j, \quad \mathcal{P}_{\mu ijkl} = \partial_\mu \phi_{ijkl} - g A_\mu^M \mathcal{P}_{Mijkl}. \quad (3.46)$$

Under the gauge transformation, \mathcal{V} changes as

$$\delta \mathcal{V} = -g \Lambda^M X_M \mathcal{V} = -2ig \Lambda^M \mathcal{V} \mathcal{V}_M^N T_N, \quad (3.47)$$

which implies

$$\delta \phi_{ijkl} = -g \Lambda^M \mathcal{P}_{Mijkl}, \quad (3.48)$$

By the $E_{7(7)}$ isometry, let O move to the extremum of the potential. It then follows that the broken gauge symmetry is described by the condition $\Lambda^M \mathcal{P}_{Mijkl} \neq 0$. We find that the corresponding $\delta \phi_{ijkl}$ are the Nambu-Goldstone bosons, which are responsible for the vector fields to acquire mass.

Let us compute the mass matrix for the Nambu-Goldstone directions. We first note that \mathcal{P}_M and \mathcal{Q}_M take the following forms at the origin,

$$\mathcal{P}_{[ab]ijkl} = \frac{1}{16} (\Gamma_{ijkl})_{c[a} \theta_{b]c}, \quad \mathcal{Q}_{[ab]i}^j = \frac{1}{4} \theta_{c[a} (\Gamma_{b]c})^{ij}, \quad (3.49)$$

where $(\Gamma^{ijkl})_{ab} = \Gamma_{ac}^{[ij} \Gamma_{cb}^{kl]}$. Converting back to the $\text{Sp}(56, \mathbb{R})$ representation via (3.18), the scalar fluctuation (3.48) can be expressed as

$$2S_{[a}^{[c} \delta_{b]}^{d]} + iU_{abcd} = \frac{1}{16} \Gamma_{ab}^{ij} \Gamma_{cd}^{kl} \delta \phi_{ijkl} \propto \Gamma_{ab}^{ij} \Gamma_{cd}^{kl} (\Gamma^{ijkl})_{p[e} \theta_{f]p} \Lambda^{[ef]} \propto (\Lambda \theta - \theta \Lambda)^{[c} \delta_{[a}^d]_{b]}, \quad (3.50)$$

where we have used

$$\Gamma_{ab}^{ij} \Gamma_{cd}^{kl} (\Gamma^{ijkl})_{pq} = 128 (\delta_{p[a} \delta_{b][c} \delta_{d]q} + \delta_{q[a} \delta_{b][c} \delta_{d]p}) + 32 \delta_{a[c} \delta_{d]b} \delta_{pq}. \quad (3.51)$$

Therefore, the Nambu-Goldstone bosons are encoded only into the S field of the form,

$$S \propto \Lambda \theta - \theta \Lambda, \quad \Lambda = -^T \Lambda, \quad U = 0. \quad (3.52)$$

Dividing Λ into n_1 and n_2 blocks

$$\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ -^T \Lambda_{12} & \Lambda_{22} \end{pmatrix}, \quad (3.53)$$

one can derive

$$S = -(\lambda_1 - \lambda_2) \begin{pmatrix} \mathbb{O}_{n_1} & \Lambda_{12} \\ ^T \Lambda_{12} & \mathbb{O}_{n_2} \end{pmatrix}, \quad (3.54)$$

and

$$M_{(1)}^2 = -\frac{1}{8} g^2 (\lambda_1 - \lambda_2)^2 (\lambda_1 + \lambda_2) [(n_1 - 2)\lambda_1 + (n_2 - 2)\lambda_2] \text{Tr} (^T \Lambda_{12} \Lambda_{12}) = 0, \quad (3.55)$$

where we inserted the extremum condition (3.7) at the last equality. Therefore, we conclude that A_{12} field corresponds to the Nambu-Goldstone bosons of the broken noncompact gauge symmetries. They would be absorbed as the longitudinal modes of gauge fields to getting massive.

4. Dyonic gaugings

We move on to the case where additional **36** charges are turned on,

$$\Theta_{ab}{}^c{}_d = \delta_{[a}{}^c \theta_{b]d}, \quad \Theta^{abc}{}_d = \delta^{[a}{}_d \xi^{b]c}, \quad (4.1)$$

where θ and ξ are (possibly noninvertible) symmetric tensors. Since both electric and magnetic charges are introduced, we shall refer to it as dyonic. The gauge generators are now given by

$$X_{[ab]} = \begin{pmatrix} X_{[ab][cd]}{}^{[ef]} & 0 \\ 0 & X_{[ab]}{}^{[cd]}{}_{[ef]} \end{pmatrix}, \quad X^{[ab]} = \begin{pmatrix} X^{[ab]}{}_{[cd]}{}^{[ef]} & 0 \\ 0 & X^{[ab][cd]}{}_{[ef]} \end{pmatrix}, \quad (4.2)$$

where

$$\begin{aligned} X_{[ab][cd]}{}^{[ef]} &= \delta_{[a}{}^{[e} \theta_{b][c} \delta_{d]}{}^{f]}, & X_{[ab]}{}^{[cd]}{}_{[ef]} &= -\delta_{[a}{}^{[c} \theta_{b][e} \delta_{f]}{}^{d]} \\ X^{[ab]}{}_{[cd]}{}^{[ef]} &= -\delta_{[c}{}^{[a} \xi^{b][e} \delta_{d]}{}^{f]}, & X^{[ab][cd]}{}_{[ef]} &= \delta_{[e}{}^{[a} \xi^{b][c} \delta_{f]}{}^{d]}. \end{aligned} \quad (4.3)$$

The value of the potential at the origin gives the cosmological constant,

$$V_c = \frac{g^2}{8} \left[\frac{1}{4} \text{Tr}(\theta^2) - \frac{1}{8} \text{Tr}(\theta)^2 + \frac{1}{4} \text{Tr}(\xi^2) - \frac{1}{8} \text{Tr}(\xi)^2 \right]. \quad (4.4)$$

4.1 Vacua

The extremum condition boils down to [20]

$$2(\theta^2 - \xi^2) - (\theta \text{Tr} \theta - \xi \text{Tr} \xi) = 2a \mathbb{I}_8, \quad (4.5)$$

where a is an arbitrary real constant. The solution for the quadratic constraint is given by

$$\xi = c\theta^{-1} \quad (c \in \mathbb{R}), \quad \text{or} \quad \xi\theta = 0. \quad (4.6)$$

These cases will be discussed separately in the following.

(I) $\theta \propto \xi^{-1}$. We start with the discussion for the case in which both θ and ξ are invertible. Letting

$$x := \frac{1}{2} \text{Tr}(\theta), \quad y := \frac{1}{2} \text{Tr}(\theta^{-1}), \quad (4.7)$$

the stationary point condition (4.5) can be equivalently written as

$$\theta^4 - x\theta^3 - a\theta^2 + c^2y\theta - c^2\mathbb{I}_8 = 0. \quad (4.8)$$

Since equation (4.8) is invariant under the similarity transformation $\theta \rightarrow P\theta P^{-1}$, we can restrict to diagonal θ . Moreover equations (4.4) and (4.8) are invariant under the rescaling $\theta \rightarrow e^\alpha \theta$ with $c \rightarrow e^{2\alpha} c, g \rightarrow e^{-\alpha} g$ ($\alpha \in \mathbb{R}$). Noticing that the embedding tensor arises together with the coupling constant, we can achieve $c = 1$ without loss of generality.

Since θ obeys a quartic polynomial, it has four eigenvalues λ_i ($i = 1, \dots, 4$) with degeneracy $n_i (\geq 0)$,

$$\theta = \lambda_1 \mathbb{I}_{n_1} \oplus \lambda_2 \mathbb{I}_{n_2} \oplus \lambda_3 \mathbb{I}_{n_3} \oplus \lambda_4 \mathbb{I}_{n_4}, \quad \sum_i n_i = 8. \quad (4.9)$$

From (4.8) one can easily derive

$$x = \sum_i \lambda_i, \quad y = - \sum_{i < j < k} \lambda_i \lambda_j \lambda_k, \quad a = - \sum_{i < j} \lambda_i \lambda_j, \quad \prod_i \lambda_i = -1. \quad (4.10)$$

Hence λ_i 's satisfying the following relation correspond to the critical point,

$$\sum_i (n_i - 2)\lambda_i = 0, \quad \sum_i \frac{n_i - 2}{\lambda_i} = 0, \quad \prod_i \lambda_i = -1 \quad (4.11)$$

Substitution of (4.11) into (4.4) yields

$$V_c = \frac{g^2}{32} \sum_i (n_i - 2)(\lambda_i^2 + \lambda_i^{-2}). \quad (4.12)$$

Therefore neither the ordering of λ_i nor the overall sign flip $\lambda_i \rightarrow -\lambda_i$ affect the scalar potential.

We are now going to classify all critical points satisfying (4.11). Letting us denote $p_i := n_i - 2$, $\sum_i p_i = 0$ and $-2 \leq p_i \leq 6$ must be satisfied. Hence there are 15 possible combinations of $\{p_i\}$, which can be categorized into the following 3 groups,

- (i) : $\{(4, -2, -2, 0), (3, -2, -1, 0), (2, -2, 0, 0), (2, -1, -1, 0), (1, 1, -2, 0), (1, -1, 0, 0), (0, 0, 0, 0)\}$,
- (ii) : $\{(2, -2, 1, -1), (1, -1, 1, -1), (2, -2, 2, -2)\}$,
- (iii) : $\{(6, -2, -2, -2), (5, -1, -2, -2), (3, 1, -2, -2), (4, -2, -1, -1), (3, -1, -1, -1)\}$.

Since the ordering of p_i 's is irrelevant, we can take (i) $p_4 = 0$, (ii) $p_1 = -p_2$ with $p_3 = -p_4$ and (iii) $p_3 = p_4$, respectively without losing generality. In the following, we shall discuss separately these cases.

(i) $p_4 = 0$. Equation (4.11) implies that all cases belonging to this family can be identified as degenerate cases of $p_i = 0$ ($i = 1, \dots, 4$). Hence the θ tensor can be written as

$$\theta = r\mathbb{I}_2 \oplus s\mathbb{I}_2 \oplus t\mathbb{I}_2 \oplus \left(-\frac{1}{rst}\right)\mathbb{I}_2, \quad \det\theta = 1, \quad (4.13)$$

where r , s and t are real parameters. We find that the cosmological constant (4.12) vanishes and one of the eigenvalues must have opposite sign from others, since the overall sign flip has no effect. Hence these vacua correspond to the $\text{SO}(6, 2)$ gauging, which spontaneously breaks down to a compact group $\text{SO}(2) \times \text{SO}(2) \times \text{SO}(2) \times \text{SO}(2)$ at the vacua. The residual gauge symmetry would be enhanced to $\text{SO}(4) \times \text{SO}(2) \times \text{SO}(2)$ for $s = r$ and to $\text{SO}(6) \times \text{SO}(2)$ for $r = s = t$.

As we have seen, these vacua are parametrized by 3 continuous parameters. It is noted that the determinant remains invariant $\det\tilde{\theta} = \det\theta$ under the $\text{SL}(8, \mathbb{R})$ transformation $\theta \rightarrow \tilde{\theta} = {}^T U \theta U$ ($\det U = 1$). If $\det\theta = \pm 1$ had not been satisfied, it would correspond to the deformation of the theory. This is not the case now, since $\det\theta = 1$ is always satisfied. This is consistent with the fact that the moduli mass matrix vanishes exactly in the directions corresponding to the variation of these parameters, as we will see later.

(ii) $p_1 = -p_2$ and $p_3 = -p_4$ ($p_i \neq 0$). In this case, $(n_1, n_2, n_3, n_4) = (4, 0, 3, 1), (3, 1, 3, 1), (4, 0, 4, 0)$ are relevant. Inserting $\lambda_4 = -1/(\lambda_1 \lambda_2 \lambda_3)$ into the first two equations of (4.11), we get two quadratic equations for λ_3 ,

$$p_3 \lambda_1 \lambda_2 \lambda_3^2 + p_1 \lambda_1 \lambda_2 (\lambda_1 - \lambda_2) \lambda_3 + p_3 = 0, \quad p_3 \lambda_1 \lambda_2 \lambda_3^2 + p_1 (\lambda_1^{-1} - \lambda_2^{-1}) \lambda_3 + p_3 = 0. \quad (4.14)$$

These equations imply $[(\lambda_1 \lambda_2)^2 + 1](\lambda_1 - \lambda_2) = 0$, giving $\lambda_2 = \lambda_1, \pm i\lambda_1$. In the $\lambda_2 = \lambda_1$ case, equation (4.14) implies that λ_i cannot be all real, so that only the $(n_1, n_2, n_3, n_4) = (4, 0, 4, 0)$ case is possible. The $\lambda_2 = \pm i/\lambda_1$ case amounts to the permutations of eigenvalues for the $\lambda_2 = \lambda_1$ case. Hence, the $(4, 0, 3, 1), (3, 1, 3, 1)$ types have no fixed points and this class of solution corresponds to the $\text{SO}(4, 4)$ dS vacua,

$$\theta = \lambda \mathbb{I}_4 \oplus (-\lambda) \mathbb{I}_4, \quad V = \frac{g^2}{4} (\lambda^2 + \lambda^{-2}), \quad \det\theta = \lambda^8 > 0, \quad \lambda \in \mathbb{R}. \quad (4.15)$$

At the vacua, the noncompact gauge symmetry is spontaneously broken to $\text{SO}(4) \times \text{SO}(4)$.

Up to this point, we are left with a single parameter λ . Usually we set $\det\theta = \pm 1$, giving $\lambda = 1$ and $V = g^2/2$. Previous studies which did not employ the embedding tensor formalism have imposed this relation, so any particular attention has been paid to this freedom. However, it appears that this remaining freedom implies that we have a one-parameter family of $\text{SO}(4, 4)$ deformed theories. This is in sharp contrast with the case (i), for which $\det\theta = 1$ is always fulfilled. Now $\det\theta = \pm 1$ is not satisfied, hence it cannot be transformed by the $\text{SL}(8, \mathbb{R})$ action to $\det\tilde{\theta} = \pm 1$, implying the deformation of the theory.

Although it is important to show which parameter region corresponds to the equivalent theories, this issue is in general difficult and beyond the scope of the present article.³ Hence, we will simply specify the allowed range of deformation parameter. However, as far as the stability issue is concerned, the mass spectrum is nevertheless insensitive to the deformation parameter as we will prove in the next subsection.

(iii) $p_3 = p_4 (\neq 0)$. We next discuss the $p_3 = p_4$ case, viz, $(n_1, n_2, n_3, n_4) = (8, 0, 0, 0), (7, 1, 0, 0), (5, 3, 0, 0), (6, 0, 1, 1)$ and $(5, 1, 1, 1)$. Inserting $\lambda_4 = -1/(\lambda_1\lambda_2\lambda_3)$ into the first two equations of (4.11), we get two quadratic equations for λ_3 ,

$$p_3\lambda_1\lambda_2\lambda_3^2 + \lambda_1\lambda_2(p_1\lambda_1 + p_2\lambda_2)\lambda_3 - p_3 = 0, \quad p_3\lambda_1\lambda_2\lambda_3^2 - \left(\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}\right)\lambda_3 - p_3 = 0, \quad (4.16)$$

Compatibility of these equations translates into a cubic equation for λ_2 ,

$$p_2\lambda_1^2\lambda_2^3 + p_1\lambda_1^3\lambda_2^2 + p_1\lambda_2 + p_2\lambda_1 = 0. \quad (4.17)$$

The solution of the above equation can be most conveniently parametrized as

$$\lambda_1 = \sqrt{-\frac{s(p_2s^2 + p_1)}{p_1s^2 + p_2}}, \quad \lambda_2 = \frac{s}{\lambda_1}, \quad (4.18)$$

where $s (\neq 0, \pm\sqrt{-p_2/p_1}, \pm\sqrt{-p_1/p_2})$ is a real parameter (it leads to the contradiction if s is complex). In this case, the cosmological constant (4.12) reduces to

$$V_c = -\frac{g^2p_1p_2(p_1 + p_2)(1 + s^2)^3}{16s(p_1s^2 + p_2)(p_2s^2 + p_1)}. \quad (4.19)$$

We now take a closer look at each vacuum.

(8,0,0,0): The θ tensor and the potential are given by

$$\theta = \lambda\mathbb{I}_8, \quad V_c = -\frac{3g^2(1 + \lambda^4)}{4\lambda^2}. \quad (4.20)$$

$\lambda \in \mathbb{R}$ is a deformation parameter. If we require $\det\theta = 1$, we have $\lambda = 1$ as usual. This is the well-known maximally supersymmetric AdS vacua at which all (pseudo)scalars vanish.

(7,1,0,0): The θ tensor and the potential are given by

$$\theta = \lambda\mathbb{I}_7 \oplus \frac{s}{\lambda}\mathbb{I}_1, \quad \lambda = \sqrt{\frac{s(s^2 - 5)}{5s^2 - 1}},$$

$$V = -\frac{5g^2(1 + s^2)^3}{4s(-5 + s^2)(-1 + 5s^2)}, \quad \det\theta = \frac{s^4(-5 + s^2)^3}{(-1 + 5s^2)^3}. \quad (4.21)$$

³Recently it has been conjectured that the different theories may be distinguished according to the eigenvalues of tensor classifier constructed from a quartic invariant of $E_{7(7)}$, in analogy with the black hole geometry [1].

s is a deformation parameter. When $0 < s < 1/\sqrt{5}$, $\sqrt{5} < s$, we have $\det\theta > 0$, producing AdS critical points of the $\text{SO}(8)$ gauging. Whereas, the parameter region $-\sqrt{5} < s < -1/\sqrt{5}$ ($\det\theta < 0$) provides AdS critical points of the $\text{SO}(7, 1)$ gauging. If we require $\det\theta = \pm 1$, the former case gives $V_c = -25\sqrt{5}g^2/32$ with $s = \sqrt{5} \pm 2$, and the latter case gives $V_c = -5g^2/8$ with $s = -1$. At these vacua, the gauge symmetries are spontaneously broken to $\text{SO}(7)$.

(5,3,0,0): The θ tensor and the potential are given by

$$\begin{aligned}\theta &= \lambda \mathbb{I}_5 \oplus \frac{s}{\lambda} \mathbb{I}_3, \quad \lambda = \sqrt{-\frac{s(3+s^2)}{3s^2+1}}, \\ V &= -\frac{3g^2(1+s^2)^3}{4s(3+s^2)(1+3s^2)}, \quad \det\theta = -\frac{s^4(3+s^2)}{1+3s^2} < 0.\end{aligned}\quad (4.22)$$

$s(< 0)$ is a deformation parameter. This case yields the dS vacua of the $\text{SO}(5, 3)$ gauging with a residual gauge symmetry $\text{SO}(5) \times \text{SO}(3)$. Assuming $\det\theta = -1$, we have $V_c = 3g^2/8$ with $s = -1$.

(6,0,1,1): We have

$$\begin{aligned}\theta &= \lambda \mathbb{I}_6 \oplus \lambda_+ \mathbb{I}_1 \oplus \lambda_- \mathbb{I}_1, \quad \lambda = \sqrt{\frac{s(s^2-2)}{2s^2-1}}, \quad \lambda_{\pm} = \pm \sigma \frac{s(\sqrt{2}s \mp 3) + \sqrt{2}}{\sqrt{s(2s^2-1)(s^2-2)}} \\ V &= -\frac{g^2(1+s^2)^3}{4s(2s^2-1)(s^2-2)}, \quad \det\theta = -\frac{s^2(-2+s^2)^3}{(-1+2s^2)^3},\end{aligned}\quad (4.23)$$

where s is a deformation parameter, $\sigma = -1$ for $0 < s < 1/\sqrt{2}$ and $\sigma = 1$ otherwise. For $-\sqrt{2} < s < -(1/\sqrt{2})$, we have $\det\theta > 0$, whereby AdS vacua of the $\text{SO}(8)$ gauging are realized. For $s > \sqrt{2}$ and $0 < s < 1/\sqrt{2}$, we have $\det\theta < 0$, corresponding to the AdS vacua of the $\text{SO}(7, 1)$ gauging. In both cases the remaining gauge symmetry is $\text{SO}(6)$. If we suppose $\det\theta = \pm 1$, $\det\theta = 1$ gives $V_c = -2g^2$ at $s = -1$, while $\det\theta = -1$ gives $V_c \sim -0.6896g^2$ at $s = [(7+3\sqrt{3} \pm \sqrt{72+42\sqrt{3}})/2]^{1/2}$. The last vacua seem new ones in the undeformed $\text{SO}(7, 1)$ gauged supergravity.

(5,1,1,1): We have

$$\begin{aligned}\theta &= \lambda \mathbb{I}_5 \oplus \frac{s}{\lambda} \oplus \lambda_+ \mathbb{I}_1 \oplus \lambda_- \mathbb{I}_1, \quad \lambda = \sqrt{\frac{s(s^2-3)}{3s^2-1}}, \quad \lambda_{\pm} = \pm \sigma \frac{s(\sqrt{3}s \mp 4) + \sqrt{3}}{\sqrt{s(s^2-3)(3s^2-1)}}, \\ V &= -\frac{3g^2(1+s^2)^3}{8s(3s^2-1)(s^2-3)}, \quad \det\theta = -\frac{s^2(-3+s^2)^2}{(1-3s^2)^2} < 0.\end{aligned}\quad (4.24)$$

$s(\neq 0, \pm 1/\sqrt{3}, \pm \sqrt{3})$ is a deformation parameter, $\sigma = -1$ for $0 < s < 1/\sqrt{3}$ and $\sigma = 1$ otherwise. This gives AdS critical points of the $\text{SO}(7, 1)$ gauging. When $\det\theta = -1$ we have $V_c = -3g^2/4$ at $s = -1, 2 \pm \sqrt{3}$. However it turns out that these cases simply relabel the eigenvalues hence give the equivalent vacua.

(II) $\theta\xi = 0$. We shall next discuss the $\theta\xi = 0$ case, in which θ and ξ can be taken to be

$$\theta = \tilde{\theta} \oplus \mathbb{O}_{n_3+n_4}, \quad \xi = \mathbb{O}_{n_1+n_2} \oplus \tilde{\xi}, \quad \sum_i n_i = 8, \quad (4.25)$$

where $\tilde{\theta}$ and $\tilde{\xi}$ are $(n_1 + n_2) \times (n_1 + n_2)$ and $(n_3 + n_4) \times (n_3 + n_4)$ matrices, respectively. These tensors give decoupled quadratic equations,

$$2\tilde{\theta}^2 - \tilde{\theta}\text{Tr}(\tilde{\theta}) = 2a\mathbb{I}_{n_1+n_2}, \quad -2\tilde{\xi}^2 + \tilde{\xi}\text{Tr}(\tilde{\xi}) = 2a\mathbb{I}_{n_3+n_4}, \quad (4.26)$$

where a is a constant. So they can be simultaneously taken to be diagonal forms,

$$\theta = \begin{pmatrix} \lambda_1 \mathbb{I}_{n_1} & & \\ & \lambda_2 \mathbb{I}_{n_2} & \\ & & \mathbb{O}_{n_3+n_4} \end{pmatrix}, \quad \xi = \begin{pmatrix} \mathbb{O}_{n_1+n_2} & & \\ & \kappa_3 \mathbb{I}_{n_3} & \\ & & \kappa_4 \mathbb{I}_{n_4} \end{pmatrix}. \quad (4.27)$$

The critical point condition (4.26) reduces to

$$\sum_{i=1}^2 (n_i - 2) \lambda_i = 0, \quad \sum_{i=3}^4 (n_i - 2) \kappa_i = 0, \quad \lambda_1 \lambda_2 = -\kappa_3 \kappa_4. \quad (4.28)$$

Using this condition, the potential is now given by

$$V_c = \frac{g^2}{32} \left[\sum_{i=1}^2 (n_i - 2) \lambda_i^2 + \sum_{i=3}^4 (n_i - 2) \kappa_i^2 \right]. \quad (4.29)$$

Since (4.26) is invariant under $\theta \rightarrow e^\alpha \theta$, $\xi \rightarrow e^\alpha \xi$ with $a \rightarrow e^{2\alpha} a$ ($\alpha \in \mathbb{R}$), only one of the eigenvalues can take any value we wish, since θ and ξ cannot be rescaled independently. Note also the invariance under $\theta \rightarrow -\theta$ and $\xi \rightarrow -\xi$, and $\theta \leftrightarrow \xi$. We will below examine the vacuum structure and the mass spectrum depending on if $n_i - 2$ is zero or not.

(i) $n_4 = 2$. In this case we can infer that the potential vanishes and the Minkowski vacua are realized. Taking the rescaling freedom into account, two kinds of gaugings are possible.

(a) One is the $\text{SO}(4) \times \text{SO}(2, 2) \ltimes \mathbb{T}^{16}$ gauging, for which

$$\theta = \begin{pmatrix} s \mathbb{I}_2 & & \\ & (1/s) \mathbb{I}_2 & \\ & & \mathbb{O}_4 \end{pmatrix}, \quad \xi = \begin{pmatrix} \mathbb{O}_4 & & \\ & t \mathbb{I}_2 & \\ & & -(1/t) \mathbb{I}_2 \end{pmatrix}, \quad (4.30)$$

where s and t are real parameters. At the vacua the gauge symmetry is broken to $\text{SO}(2) \times \text{SO}(2) \times \text{SO}(2)$.

(b) The other is the $\text{SO}(2) \times \text{SO}(2) \ltimes \mathbb{T}^{20}$ gauging, for which

$$\theta = \begin{pmatrix} \mathbb{I}_2 & \\ & \mathbb{O}_6 \end{pmatrix}, \quad \xi = \begin{pmatrix} \mathbb{O}_6 & \\ & s \mathbb{I}_2 \end{pmatrix}, \quad (4.31)$$

where s is a real parameter. The residual gauge symmetry is $\text{SO}(2) \times \text{SO}(2)$.

(ii) $n_i \neq 2$. Assuming $n_i \neq 2$, we can obtain

$$\lambda_2 = -\frac{n_1 - 2}{n_2 - 2} \lambda_1, \quad \kappa_3 = \sqrt{-\frac{(n_1 - 2)(n_4 - 2)}{(n_2 - 2)(n_3 - 2)}} \lambda_1, \quad \kappa_4 = -\frac{n_3 - 2}{n_4 - 2} \kappa_3. \quad (4.32)$$

The possible values of $n_i (\neq 2)$ and the corresponding gaugings are given by

$$(7, 0, 1, 0) : \text{SO}(7) \ltimes \mathbb{T}^7, \quad (4.33a)$$

$$(6, 1, 0, 1) : \text{SO}(7) \ltimes \mathbb{T}^7, \quad (4.33b)$$

$$(6, 0, 1, 1) : \text{SO}(6) \times \text{SO}(1, 1) \ltimes \mathbb{T}^{12}, \quad (4.33c)$$

$$(5, 1, 1, 1) : \text{SO}(6) \times \text{SO}(1, 1) \ltimes \mathbb{T}^{12}. \quad (4.33d)$$

Otherwise, eigenvalues will be imaginary. The potential now reads

$$V_c = \frac{g^2(n_1 - 2)}{16(n_2 - 2)}(n_1 + n_2 - 4)\lambda_1^2. \quad (4.34)$$

One can easily verify that all vacua falling into this family correspond to AdS. Eliminating λ_1 by the rescaling freedom, no tunable parameters are left and we arrive at the following exhaustive list.

(7,0,1,0): We have $\text{SO}(7) \ltimes \mathbb{T}^7$ gauging with

$$\theta = \mathbb{I}_7 \oplus 0, \quad \xi = \mathbb{O}_7 \oplus (\sqrt{5}), \quad V_c = -\frac{15}{32}g^2. \quad (4.35)$$

The gauge symmetry is broken to $\text{SO}(7)$.

(6,1,0,1): We have $\text{SO}(7) \ltimes \mathbb{T}^7$ gauging with

$$\theta = \mathbb{I}_6 \oplus 4 \oplus 0, \quad \xi = \mathbb{O}_7 \oplus (-2\sqrt{2}), \quad V_c = -\frac{3}{4}g^2. \quad (4.36)$$

The gauge symmetry is broken to $\text{SO}(6)$.

(6,0,1,1): We have $\text{SO}(6) \times \text{SO}(1,1) \ltimes \mathbb{T}^{12}$ gauging with

$$\theta = \mathbb{I}_6 \oplus \mathbb{O}_2, \quad \xi = \mathbb{O}_6 \oplus (\sqrt{2}) \oplus (-\sqrt{2}), \quad V_c = -\frac{1}{4}g^2. \quad (4.37)$$

The gauge symmetry is broken to $\text{SO}(6)$.

(5,1,1,1): We have $\text{SO}(6) \times \text{SO}(1,1) \ltimes \mathbb{T}^{12}$ gauging with

$$\theta = \mathbb{I}_5 \oplus (3) \oplus \mathbb{O}_2, \quad \xi = \mathbb{O}_6 \oplus (\sqrt{3}) \oplus (-\sqrt{3}), \quad V_c = -\frac{3}{8}g^2. \quad (4.38)$$

The gauge symmetry is broken to $\text{SO}(5)$.

4.2 Mass spectrum

Inserting (3.9) and (4.2) into (2.22), we can show after a rather lengthy but straightforward computation that the mass spectra for S and U are the simple sum of θ and ξ terms,

$$M_{(1)}^2 = M_{(1)}^2(\theta) + M_{(1)}^2(\xi), \quad M_{(2)}^2 = M_{(2)}^2(\theta) + M_{(2)}^2(\xi), \quad (4.39)$$

where $M_{(i)}(\theta)$ is given by (3.13) and (3.14). When θ has a structure of n_1 and n_2 blocks only (i.e., $n_3 = n_4 = 0$), it turns out that equations (3.24) and (3.34) continue to hold in the dyonic case because $M_{(i)}^2$ and V_c enjoy the property that θ and ξ terms are decoupled, so that they sum up to give the same contributions.

We give general formulas applying to all dyonic cases. Following the same argument in the preceding section, we decompose θ and ξ into $n_1 + n_2 + n_3 + n_4$ blocks,

$$\theta = \begin{pmatrix} \lambda_1 \mathbb{I}_{n_1} & & & \\ & \lambda_2 \mathbb{I}_{n_2} & & \\ & & \lambda_3 \mathbb{I}_{n_3} & \\ & & & \lambda_4 \mathbb{I}_{n_4} \end{pmatrix}, \quad \xi = \begin{pmatrix} \kappa_1 \mathbb{I}_{n_1} & & & \\ & \kappa_2 \mathbb{I}_{n_2} & & \\ & & \kappa_3 \mathbb{I}_{n_3} & \\ & & & \kappa_4 \mathbb{I}_{n_4} \end{pmatrix}. \quad (4.40)$$

and correspondingly

$$S = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix}, \quad (4.41)$$

where A_{ij} is an $n_i \times n_j$ matrix satisfying

$$A_{ij} = {}^T A_{ji}, \quad \sum_i n_i A_{ii} = 0. \quad (4.42)$$

Denoting $A_{ii} = \hat{A}_i + (1/n_i)\text{Tr}(A_{ii})$, the mass matrix for S is given by

$$M_{(1)}^2 = \sum_{i,j} \mu_{ij}^2 \text{Tr}(A_{ii}) \text{Tr}(A_{jj}) + \sum_i m_i^2 \text{Tr}(\hat{A}_i^2) + 2 \sum_{i < j} m_{ij*}^2 \text{Tr}({}^T A_{ij} A_{ij}), \quad (4.43)$$

with

$$\mu_{ij}^2 = \frac{g^2}{8} \left[\frac{1}{n_i} \left\{ 4(\lambda_i^2 + \kappa_i^2) - \lambda_i \sum_k n_k \lambda_k - \kappa_i \sum_k n_k \kappa_k \right\} \delta_{ij} - (\lambda_i \lambda_j + \kappa_i \kappa_j) \right], \quad (4.44a)$$

$$m_i^2 = \frac{g^2}{8} \left[4(\lambda_i^2 + \kappa_i^2) - \lambda_i \sum_j n_j \lambda_j - \kappa_i \sum_j n_j \kappa_j \right], \quad (4.44b)$$

$$m_{ij*}^2 = \frac{g^2}{8} \left[-\frac{1}{2}(\lambda_i + \lambda_j) \sum_k n_k \lambda_k + (\lambda_i + \lambda_j)^2 - \frac{1}{2}(\kappa_i + \kappa_j) \sum_k n_k \kappa_k + (\kappa_i + \kappa_j)^2 \right] = 0. \quad (4.44c)$$

In the last step, we have used the critical point conditions (4.11) and (4.28). Hence A_{ij} ($i < j$) field is always massless. Repeating the parallel argument in section 3.3, we can find that this direction corresponds to the Nambu-Goldstone bosons.

In order to achieve the canonical normalization of S , we have to eliminate $\text{Tr}(A_{44})$ by the condition $\text{Tr}(S) = 0$. After some simple algebra, we obtain the diagonal form,

$$\frac{1}{2}(\partial \text{Tr}(S))^2 = \frac{1}{2} \left[\sum_{i=1}^4 (\partial \text{Tr}(\hat{A}_i))^2 + 2 \sum_{i < j} \text{Tr}(\partial A_{ij} \partial^T A_{ij}) + \sum_{i=1}^3 (\partial a_i)^2 \right], \quad (4.45)$$

where a_i 's are canonically normalized scalars and defined by

$$a_1 := \sqrt{\frac{n_1 + n_2 + n_3 + n_4}{n_1(n_2 + n_3 + n_4)}} \text{Tr}(A_{11}), \quad (4.46a)$$

$$a_2 := \sqrt{\frac{n_2 + n_3 + n_4}{n_2(n_3 + n_4)}} \left[\text{Tr}(A_{22}) + \frac{n_2}{n_2 + n_3 + n_4} \text{Tr}(A_{11}) \right], \quad (4.46b)$$

$$a_3 := \sqrt{\frac{n_3 + n_4}{n_3 n_4}} \left[\text{Tr}(A_{33}) + \frac{n_3}{n_3 + n_4} (\text{Tr}(A_{11}) + \text{Tr}(A_{22})) \right]. \quad (4.46c)$$

Note that a_3 should be absent when $n_4 = 0$. Since the mass term for $\text{Tr}(\hat{A}_i^2)$ and a_i do not have illuminating expressions in general if we eliminate $\text{Tr}(A_{44})$, we examine these terms for each case. As it turns out, however, we see that the above choice (4.46) always leads to the diagonal mass matrix for the trace part.

Next, we shall determine the eigenvalue of U . To this end, we classify the basis of self-dual 4-form U in terms of the degeneracy of eigenvalues (λ_i, κ_i) . Let us define

$$\vec{\ell} := (\ell_1, \dots, \ell_m), \quad \ell_i \leq n_i, \quad \sum_i^m \ell_i = 4 \quad m \leq 4, \quad (4.47)$$

where ℓ_i 's are nonnegative integers. The multiplicities belonging to the same $\vec{\ell}$ are given by

$$n_1 C_{\ell_1} \times \cdots \times n_m C_{\ell_m} : \quad \ell_1 > n_1/2 \quad \text{or} \quad \ell_1 = n_1/2, \ell_2 > n_2/2 \quad \text{or} \cdots, \\ \ell_1 = n_1/2, \ell_2 = n_2/2, \dots, \ell_m > n_m/2. \quad (4.48a)$$

$$\frac{1}{2} n_1 C_{n_1/2} \times \cdots \times n_m C_{n_m/2} : \quad \ell_i = n_i/2 \quad (i = 1, \dots, m). \quad (4.48b)$$

We can easily verify

$$(1 + \star)(\theta^r {}_{[a} \theta^s {}_{b} + \kappa^r {}_{[a} \kappa^s {}_{b}) U_{cd]rs} = 2\mu_{\vec{\ell}} U_{abcd}, \quad (4.49)$$

where

$$\mu_{\vec{\ell}} = \frac{1}{24} \left[\left(\sum_i \ell_i \lambda_i \right)^2 + \left(\sum_i (n_i - \ell_i) \lambda_i \right)^2 + \left(\sum_i \ell_i \kappa_i \right)^2 + \left(\sum_i (n_i - \ell_i) \kappa_i \right)^2 - \sum_i n_i (\lambda_i^2 + \kappa_i^2) \right]. \quad (4.50)$$

Hence we obtain the mass eigenvalues

$$M_{(2)}^2 = \frac{1}{6} m_{[\vec{\ell}]}^2 U \cdot U, \quad (4.51)$$

with

$$m_{[\vec{\ell}]}^2 = \frac{g^2}{32} \left[2 \sum_i n_i (\lambda_i^2 + \kappa_i^2) - \left(\sum_i \ell_i \lambda_i \right)^2 - \left(\sum_i (n_i - \ell_i) \lambda_i \right)^2 - \left(\sum_i \ell_i \kappa_i \right)^2 - \left(\sum_i (n_i - \ell_i) \kappa_i \right)^2 \right], \quad (4.52)$$

which is specified by nonnegative integers ℓ_i satisfying

$$0 \leq \ell_i \leq n_i, \quad \sum_i^m \ell_i = 4. \quad (4.53)$$

Since the kinetic term for scalars is given by (3.19), $m_{[\vec{\ell}]}^2$ denotes the canonical mass eigenvalues.

We are now armed with necessary tools to demonstrate mass spectra in the dyonic case.

(I) $\theta \propto \xi^{-1}$. Let us begin with the $\theta \propto \xi^{-1}$ case.

(i) $n_i = 2$. This case corresponds to the Minkowski vacua of $\text{SO}(6, 2)$ gauging, which spontaneously breaks down to $\text{SO}(2) \times \text{SO}(2) \times \text{SO}(2) \times \text{SO}(2)$. Taking the θ tensor as (4.13), equations (4.43) and (4.46) yield

$$m_{(\mathbf{1}, \mathbf{1}, \mathbf{1})(\times 3)}^2 = 0, \quad m_{*(\times 24)}^2 = 0 \quad : (\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}) + \cdots, \\ m_{i(\times 8)}^2 = \frac{g^2}{16r^2 s^2 t^2} \times \begin{cases} 4st(r-s)(r-t)(1+r^2 st), & : (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \\ 4rt(s-r)(s-t)(1+rs^2 t), & : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \\ 4rs(r-t)(s-t)(1+rst^2), & : (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \\ 4(1+r^2 st)(1+rs^2 t)(1+rst^2), & : (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}) \end{cases}. \quad (4.54)$$

From (4.52), the mass eigenvalues for pseudoscalars are given by

$$m_{(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\times 3)}^2 = 0 \quad : [2, 2, 0, 0] + [2, 0, 2, 0] + [2, 0, 0, 2],$$

$$m_{[\ell]}^2 = \frac{g^2}{16r^2s^2t^2} \times \begin{cases} t^2(r-s)^2(1+r^2s^2), & : (\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}) = [1, 1, 2, 0]_{(\times 4)} \\ s^2(r-t)^2(1+r^2t^2), & : (\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1}) = [1, 2, 1, 0]_{(\times 4)} \\ r^2(s-t)^2(1+s^2t^2), & : (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) = [2, 1, 1, 0]_{(\times 4)} \\ (1+s^2t^2)(1+sr^2t)^2, & : (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{2}) = [1, 2, 0, 1]_{(\times 4)} \\ (1+s^2r^2)(1+str^2)^2, & : (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}) = [2, 0, 1, 1]_{(\times 4)} \\ (1+r^2t^2)(1+s^2rt)^2, & : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}) = [2, 1, 0, 1]_{(\times 4)} \\ (1+s^2t^2)(1+r^2s^2)(1+t^2r^2), & : (\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2})^+ = [1, 1, 1, 1]_{(\times 8)} \end{cases} \quad (4.55)$$

It is emphasized that mass eigenvalues for the pseudoscalars are always nonnegative, whereas those for scalars are not. This implies that vacua of generic $\text{SO}(6, 2)$ gauging are unstable [23]. In the special case where (r, s, t) are pairwise equal, all mass eigenvalues become nonnegative, corresponding to the stable Minkowski vacua found in [20].

(ii) $p_3 = -p_4 (\neq 0)$. This case corresponds to the $\text{SO}(4, 4)$ dS vacua. From (3.23) and (3.32), we obtain

$$m_{0(\mathbf{1},\mathbf{1})}^2 = -2V_c, \quad m_{1(\mathbf{9},\mathbf{1})}^2 = m_{2(\mathbf{1},\mathbf{9})}^2 = 2V_c, \quad m_{*(\mathbf{4},\mathbf{4})}^2 = 0, \\ m_{[\ell=2](\times 18)}^2 = m_{(\mathbf{6},\mathbf{6})^+}^2 = 2V_c, \quad m_{[\ell=3](\times 16)}^2 = m_{(\mathbf{4},\mathbf{4})}^2 = V_c, \quad m_{[\ell=4](\times 1)}^2 = m_{(\mathbf{1},\mathbf{1})}^2 = -2V_c. \quad (4.56)$$

The tachyonic modes emerge from the $\text{SO}(4) \times \text{SO}(4)$ invariant sector. These spectra agree with the electric case.

(iii) $p_3 = -p_4 (\neq 0)$. In this case the cosmological constant is nonvanishing. Using the expression (4.19), the mass term (4.43) for S considerably simplifies to

$$\sum_{i,j} \mu_{ij}^2 \text{Tr}(A_{ii}) \text{Tr}(A_{jj}) = -2V_c \sum_{i=1}^3 a_i^2, \quad m_i^2 = \frac{4}{n_i - 2} V_c. \quad (4.57)$$

where canonical scalars a_i were defined in (4.46). As we already explained, A_{ij} ($i < j$) do not contribute to the mass term. In the case of dS vacua ($V_c > 0$), the scalars a_i are always tachyonic. The above equation exhibits that the mass spectrum is determined only by n_i , which controls the residual gauge symmetry. It also illustrates that the mass spectrum is completely independent of the deformation parameter s when normalized by the cosmological constant. In other words the “slow-roll matrix” $\eta = \partial_\rho \partial_\sigma V / V$ does not depend on s , albeit the change of the value of potential. We can also find that the mass spectrum for U shares this property.

(8,0,0,0): This case corresponds to the maximally supersymmetric AdS vacua of the $\text{SO}(8)$ gauging. The θ tensor and the potential are given by (4.20). The formulas (3.23) and (3.32) yield

$$m_{1(\mathbf{35})}^2 = -\frac{2}{3}|V_c|, \quad m_{[\ell=4](\mathbf{35})}^2 = -\frac{2}{3}|V_c|. \quad (4.58)$$

Since all of these mass eigenvalues are above the Breitenlohner-Freedman bound [29], the vacuum is stable as expected from supersymmetry.

(7,1,0,0): This gives AdS vacua of $\text{SO}(8)$ and $\text{SO}(7, 1)$ gaugings with an $\text{SO}(7)$ residual gauge symmetry. The θ tensor and the potential are given by (4.21). From (3.23) and (3.32), the mass spectrum now reads

$$m_{0(\mathbf{1})}^2 = 2|V_c|, \quad m_{1(\mathbf{27})}^2 = -\frac{4}{5}|V_c|, \quad m_{[\ell=4](\mathbf{35})}^2 = -\frac{2}{5}|V_c|. \quad (4.59)$$

$m_{*(7)}^2 = 0$ correspond to the Nambu-Goldstone fields.

(5,3,0,0): This case corresponds to the $\text{SO}(5,3)$ gauging dS vacua with a residual gauge symmetry $\text{SO}(5) \times \text{SO}(3)$. The θ tensor and the potential are given by (4.22). Equations (3.23) and (3.32) yield

$$\begin{aligned} m_{0(1,1)}^2 &= -2V_c, & m_{1(14,1)}^2 &= \frac{4}{3}V_c, & m_{2(1,5)}^2 &= 4V_c, \\ m_{[\ell=3](\times 30)}^2 &= m_{(10,3)}^2 = 2V_c, & m_{[\ell=4](\times 5)}^2 &= m_{(5,1)}^2 = -\frac{2}{3}V_c. \end{aligned} \quad (4.60)$$

$m_{*(5,3)}^2 = 0$ correspond to the Nambu-Goldstone fields.

Let us compare this with the result in [9, 11], where the $\text{SO}(5)$ invariant scalars were analyzed. The corresponding potential was shown to be

$$V = \frac{g^2}{8} \left(\frac{u^3 v^3}{w^3} + \frac{10uv}{w} - 2uvw^3 + \text{two cyclic perm.} - \frac{15}{uvw} \right), \quad (4.61)$$

where u , v and w are $\text{SO}(5)$ invariant (unnormalized) scalars. The dS vacuum $V_c = 2 \times 3^{1/4} g^2$ exists at $u = v = w = 3^{-1/4}$ (observe that the normalization of g is different from the present paper). From the expression of scalar kinetic term in reference [9, 11], one can find the canonically normalized scalars Φ_i ($i = 1, 2, 3$) as

$$\Phi_1 = \sqrt{\frac{5}{6}} \ln(uvw), \quad \Phi_2 = \frac{1}{\sqrt{6}} \ln\left(\frac{v^2}{uw}\right), \quad \Phi_3 = \frac{1}{\sqrt{2}} \ln\left(\frac{w}{u}\right). \quad (4.62)$$

Then the mass eigenvalues are given by

$$m_{\Phi_1}^2 = -2V_c, \quad m_{\Phi_2}^2 = m_{\Phi_3}^2 = 4V_c. \quad (4.63)$$

These spectra coincide with the result obtained above, where $\text{SO}(5)$ invariant scalars descend from $\text{Tr}(A_{11})$ and $\text{Tr}(\hat{A}_2^2)$.

(6,0,1,1): This case gives the AdS vacua of $\text{SO}(8)$ and $\text{SO}(7,1)$ gaugings with the residual gauge symmetry $\text{SO}(6)$. The θ tensor and the potential are given by (4.23). Now the formula (4.57) can be applied to give

$$m_{0(1)\times 2}^2 = 2|V_c|, \quad m_{1(20)}^2 = -|V_c|, \quad m_{*(6)(\times 2)}^2 = m_{*(1)}^2 = 0. \quad (4.64)$$

For the U field, equation (4.52) yields

$$m_{[4,0,0](15)}^2 = 0, \quad m_{[3,1,0](20)}^2 = -\frac{1}{4}|V_c|. \quad (4.65)$$

(5,1,1,1): We have the AdS critical point of $\text{SO}(7,1)$ gauging with the residual gauge symmetry $\text{SO}(5)$. The θ tensor and the potential are given by (4.23). Using (4.57) we obtain

$$m_{0(1)(\times 3)}^2 = 2|V_c|, \quad m_{1(14)}^2 = -\frac{4}{3}|V_c|, \quad m_{*(5)(\times 3)}^2 = m_{*(1)(\times 3)}^2 = 0. \quad (4.66)$$

From (4.52), the mass eigenvalues for U are given by

$$m_{[3,1,0,0](10)}^2 = m_{[3,0,1,0](10)}^2 = m_{[3,0,0,1](10)}^2 = 0, \quad m_{[4,0,0,0](5)}^2 = \frac{2}{3}|V_c|. \quad (4.67)$$

(II) $\theta\xi = 0$. We next turn to study the $\theta\xi = 0$ case.

(i-a) For the $\text{SO}(4) \times \text{SO}(2, 2) \ltimes \mathbb{T}^{16}$ gauging with a residual symmetry $\text{SO}(4) \times \text{SO}(2) \times \text{SO}(2)$, θ and ξ are given by (4.30) and the Minkowski vacua are realized. The mass spectra are given by

$$m_{0(\mathbf{1}, \mathbf{1}, \mathbf{1})(\times 3)}^2 = 0, \quad m_{*(\times 24)}^2 = 0 \quad : (\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}) + \dots, \\ m_i^2 = \frac{g^2}{16s^2t^2} \times \begin{cases} -4s^2t^2(1-s^2), & : (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \\ 4t^2(1-s^2), & : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \\ 4s^2t^2(1+t^2), & : (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \\ 4s^2(1+t^2), & : (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}) \end{cases}. \quad (4.68)$$

For the pseudoscalars, we have

$$m_{(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(\times 3)}^2 = 0 \quad : [2, 2, 0, 0] + [2, 0, 2, 0] + [2, 0, 0, 2], \\ m_{[\bar{\ell}]}^2 = \frac{g^2}{16s^2t^2} \times \begin{cases} t^2(1+s^2t^2), & : (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) = [2, 1, 1, 0]_{(\times 4)} \\ (s^2+t^2), & : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}) = [2, 1, 0, 1]_{(\times 4)} \\ s^2(1+s^2t^2), & : (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{2}) = [1, 2, 0, 1]_{(\times 4)} \\ s^2t^2(s^2+t^2), & : (\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1}) = [1, 2, 1, 0]_{(\times 4)} \\ t^2(1-s^2)^2, & : (\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}) = [1, 1, 2, 0]_{(\times 4)} \\ s^2(1+t^2)^2, & : (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}) = [2, 0, 1, 1]_{(\times 4)} \\ (s^2+t^2)(1+s^2t^2), & : (\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2})^+ = [1, 1, 1, 1]_{(\times 8)} \end{cases}. \quad (4.69)$$

The pseudoscalars are all stable, whereas the scalars are unstable unless $s = 1$.

(i-b) For the $\text{SO}(2) \times \text{SO}(2) \ltimes \mathbb{T}^{20}$ gauging with a residual symmetry $\text{SO}(2) \times \text{SO}(2)$, θ and ξ are given by (4.31) and the Minkowski vacua are realized. The mass spectra are

$$m_{0(\mathbf{1}, \mathbf{1})(\times 2)}^2 = 0, \quad m_i^2 = \frac{1}{4}g^2 \times \begin{cases} 1 & : (\mathbf{2}, \mathbf{1}) \\ s^2 & : (\mathbf{1}, \mathbf{2}) \\ 0 & : (\mathbf{1}, \mathbf{1})_{(\times 9)} \end{cases}, \\ m_{*(\times 20)}^2 = 0 \quad : (\mathbf{2}, \mathbf{1})_{(\times 4)} + (\mathbf{1}, \mathbf{2})_{(\times 4)} + (\mathbf{2}, \mathbf{2}), \quad (4.70)$$

and

$$m_{(\mathbf{1}, \mathbf{1})(\times 7)}^2 = 0 \quad : [2, 2, 0] + [2, 0, 2]_{(\times 6)}, \\ m_{[\bar{\ell}]}^2 = \frac{1}{16}g^2 \times \begin{cases} 1, & (\mathbf{2}, \mathbf{1})_{(\times 4)} = [1, 2, 1]_{(\times 8)} \\ s^2, & (\mathbf{1}, \mathbf{2})_{(\times 4)} = [2, 1, 1]_{(\times 8)} \\ (1+s^2), & (\mathbf{2}, \mathbf{2})_{(\times 6)}^+ = [1, 1, 2]_{(\times 12)} \end{cases} \quad (4.71)$$

Hence these vacua are stable.

(ii) $n_i \neq 2$. The AdS vacua are realized in this family. Using (4.46), (4.28) and (4.29) with $\lambda_3 = \lambda_4 = \kappa_1 = \kappa_2 = 0$, the mass spectrum of S is given by

$$M_{(1)}^2 = |V_c| \left[-\sum_{i=1}^4 \frac{4}{n_i - 2} \text{Tr}(\hat{A}_i^2) + 2 \sum_{i=1}^3 a_i^2 \right]. \quad (4.72)$$

It is gratifying that this expression accords precisely with (4.57), for which $\xi \propto \theta^{-1}$. A_{ij} are always massless irrespective of the gaugings. The above equation also confirms that the mass spectrum for S is only dependent on n_i 's, i.e., the residual gauge symmetry only. The same is true for the U field.

(7,0,1,0): This corresponds to the $\text{SO}(7) \ltimes \mathbb{T}^7$ gauging with an $\text{SO}(7)$ remaining symmetry, where θ and ξ are given by (4.35). We obtain

$$m_{0(1)}^2 = 2|V_c|, \quad m_{1(27)}^2 = -\frac{4}{5}|V_c|, \quad m_{*(7)}^2 = 0, \quad (4.73)$$

and

$$m_{[4,0](35)}^2 = -\frac{2}{5}|V_c|. \quad (4.74)$$

(6,1,0,1): We have the $\text{SO}(7) \ltimes \mathbb{T}^7$ gauging with an $\text{SO}(6)$ remaining symmetry, where θ and ξ are given by (4.36). We obtain

$$m_{0(1)(\times 2)}^2 = 2|V_c|, \quad m_{1(20)}^2 = -|V_c|, \quad m_{*(6)(\times 2)}^2 = m_{*(1)}^2 = 0, \quad (4.75)$$

and

$$m_{[3,1,0](20)}^2 = -\frac{1}{4}|V_c|, \quad m_{[4,0,0](15)}^2 = 0. \quad (4.76)$$

(6,0,1,1): We have the $\text{SO}(6) \times \text{SO}(1,1) \ltimes \mathbb{T}^{12}$ gauging with an $\text{SO}(6)$ remaining symmetry, where θ and ξ are given by (4.37). We obtain

$$m_{0(1)(\times 2)}^2 = 2|V_c|, \quad m_{1(20)}^2 = -|V_c|, \quad m_{*(6)(\times 2)}^2 = m_{*(1)}^2 = 0, \quad (4.77)$$

for scalars and

$$m_{[3,1,0](20)}^2 = -\frac{1}{4}|V_c|, \quad m_{[4,0,0](15)}^2 = 0. \quad (4.78)$$

for pseudoscalars. These are the same as (6,1,1,0) type since the residual gauge symmetries are equivalent.

(5,1,1,1): We have the $\text{SO}(6) \times \text{SO}(1,1) \ltimes \mathbb{T}^{12}$ gauging with an $\text{SO}(6)$ remaining symmetry, where θ and ξ are given by (4.38). We obtain

$$m_{0(1)(\times 3)}^2 = 2|V_c|, \quad m_{1(14)}^2 = -\frac{4}{3}|V_c|, \quad m_{*(5)(\times 3)}^2 = m_{*(1)(\times 3)}^2 = 0, \quad (4.79)$$

and

$$m_{[3,1,0,0](10)}^2 = m_{[3,0,1,0](10)}^2 = m_{[3,0,0,1](10)}^2 = 0, \quad m_{[4,0,0,0](5)}^2 = \frac{2}{3}|V_c|. \quad (4.80)$$

We enumerate the result obtained in this section in table 4.2. Except for the maximally supersymmetric AdS vacua, supersymmetries are broken completely. One can inspect that at the non-supersymmetric AdS vacua, the S field does not respect the Breitenlohner-Freedman bound, implying the linear instability of these vacua. For the Minkowski vacua, the mass spectrum for S is not necessarily positive, implying the instability.

5. Concluding remarks

We have studied the critical points and their mass spectra in maximal gauged supergravity. Although the maximal supergravity is not entitled to a unified framework for gauge interactions, scanning vacua in this theory certainly serves as a foundation for the realistic construction of string

| Gauging | G_{reg} | Λ | m_S^2 | m_U^2 |
|--|----------------------|-----------|---|---|
| SO(4, 4) | SO(4) \times SO(4) | dS | $-2_{(\times 1)}, 2_{(\times 18)}, 0_{(\times 16)}$ | $2_{(\times 18)}, 1_{(\times 16)}, -2_{(\times 1)}$ |
| SO(5, 3) | SO(5) \times SO(3) | dS | $-2_{(\times 1)}, \frac{4}{3}_{(\times 14)}, 4_{(\times 5)}, 0_{(\times 15)}$ | $2_{(\times 30)}, -\frac{2}{3}_{(\times 5)}$ |
| SO(8) SO(7, 1) SO(7) \ltimes \mathbb{T}^7 | SO(7) | AdS | $2_{(\times 1)}, -\frac{4}{5}_{(\times 27)}, 0_{(\times 7)}$ | $-\frac{2}{5}_{(\times 35)}$ |
| SO(8) SO(7, 1) SO(7) \ltimes \mathbb{T}^7 SO(6) \times SO(1, 1) \ltimes \mathbb{T}^{12} | SO(6) | AdS | $-1_{(\times 20)}, 2_{(\times 2)}, 0_{(\times 13)}$ | $0_{(\times 15)}, -\frac{1}{4}_{(\times 20)}$ |
| SO(7, 1) SO(6) \times SO(1, 1) \ltimes \mathbb{T}^{12} | SO(5) | AdS | $-\frac{4}{3}_{(\times 14)}, 2_{(\times 3)}, 0_{(\times 18)}$ | $\frac{2}{3}_{(\times 5)}, 0_{(\times 30)}$ |
| SO(6, 2) | SO(2) ⁴ | Mink. | Eq. (4.54) | Eq. (4.55) |
| SO(4) \times SO(2, 2) \ltimes \mathbb{T}^{16} | SO(2) ⁴ | Mink. | Eq. (4.68) | Eq. (4.69) |
| SO(2) \times SO(2) \ltimes \mathbb{T}^{20} | SO(2) ² | Mink. | Eq. (4.70) | Eq. (4.71) |

Table 2: Mass spectrum for dyonic gaugings. Except for the Minkowski vacua, mass eigenvalues are normalized by the absolute value of cosmological constant. Supersymmetries are completely broken.

vacua, due to the restrictive property of maximal supergravity. In particular, the result of this vacuum search can have significant implications to the construction of inflationary universe models on the base of string/M theory, because the maximal gauged supergravity may describe the gravity sector very well, including non-perturbative effects in the 10/11-dimensional framework. In addition it is also useful for the phenomenological applications to the condensed matter physics.

Utilizing the fact that the scalar fields parametrize the homogeneous space, we can analyze the 70 scalar mass spectrum at the origin of scalar space as argued in [20, 21]. Specializing to the gaugings contained in $\text{SL}(8, \mathbb{R})$, we were able to enumerate all the possible vacua. We also developed a new formulation which allows us to obtain the analytic expression of mass spectra in terms of eigenvalues of the embedding tensor. We established an interesting structure about the moduli space of vacua: when the cosmological constant is nonvanishing, the mass spectrum is only sensitive to the residual gauge symmetry at the vacua. Namely, the mass spectra for the $\text{SL}(8, \mathbb{R})$ -type gaugings have to coincide among the different theories as long as their residual gauge symmetries are identical. This resolved the issue which remained open in [20].

In some cases of dyonic gaugings, we are left with a deformation parameter s . It turns out that the mass spectrum is nevertheless insensitive to the parameter s in units of the cosmological constant. This means that SO(4, 4) and SO(5, 3) dS maxima do not provide sufficient e -foldings in the standard potential-driven inflation scenario even in the deformed theory, since the slow-roll parameter η is of order unity. We can also verify that the fraction of residual supersymmetries is not dependent on the deformation parameter, i.e., all vacua except the maximally supersymmetric AdS totally break supersymmetries.

We have also shown that the generic Minkowski vacua found in this paper do not have stable mass spectra unless the remaining continuous parameters are finely tuned. This is consistent with the result in [23].

The obvious next step is to explore the vacuum classifications for gaugings contained in other subgroup of $E_{7(7)}$, such as $E_{6(6)}$ and $\text{SU}^*(8)$. We believe that the techniques developed in this paper could be used in other frames. It is interesting to see whether the characteristic features exposed here are universal, i.e., whether the mass spectrum is insensitive to the deformation parameter and

only dependent on the residual gauge symmetry.

Another possible future work is to work out inflationary models in the maximal theory. As we have demonstrated systematically, gaugings contained in the $SL(8, \mathbb{R})$ frame fail to have stable dS vacua and the slow-roll condition is never satisfied. Even though a simple hill-top type inflation does not work, there remains a possibility for a realization of sufficient inflation around these dS saddle points with the aid of other fields. We have 35 scalars and 35 pseudoscalars, which may be able to realize quasi-dS phase if flux is turned on appropriately. We will report this issue elsewhere.

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